# PAPER : DU 2014
## PHYSICS-PH

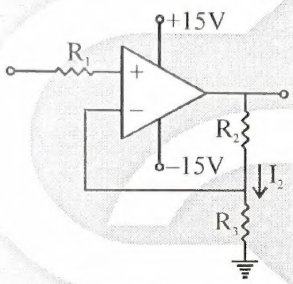
**INSTRUCTIONS:**

(*i*) This test booklet contains 75 Objective type questions.

(*ii*) Each question carries **4 marks. 1 mark** will be deducted for each incorrect answer. Only one box should be marked and the marking of more than one box will be treated as wrong answer.

## OBJECTIVE QUESTIONS

1. The non-inverting amplifier shown in the following circuit uses a 741 Op-Amp with $R_1 = 1K\Omega, R_2 = 39K\Omega, R_3 = 1K\Omega$. The typical parameter values for the 741 Op-Amp are: Open loop gain $A_v = 2 \times 10^5$, Input resistance $R_i = 2M\Omega$ and Output resistance $R_o = 75\Omega$. The Input resistance and Output resistance for the non-inverting amplifier respectively will be about:




(a) $400G\Omega, 0.015\Omega$

(b) $10G\Omega, 0.015\Omega$

(c) $400G\Omega, 0.00037\Omega$

(d) $10G\Omega, 0.00037\Omega$

2. The maximum efficiency of an npn BJT amplifier is about $\pi/4$ under the:

(a) Class A configuration (b) Class B push-pull configuration

(c) Class C tuned configuration (d) Class AB configuration

3. The *n*-channel JFET having pinch off voltage of –5V shows a transconductance of 1mA/V when the applied gate to source voltage ($V_{GS}$) is –3V. The maximum value of transconductance (in mA/V) for JFET will be:

(a) 1.5 (b) 2.0 (c) 2.5 (d) 3.0

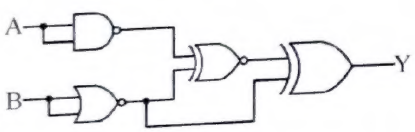
4. The minimum number of storage elements needed for the designing of a bi-stable multivibrator is:

(a) 0 (b) 1 (c) 2 (d) 3

5. The simplified Boolean expression for $Y = \Sigma_m(0,2,4,5,8,10) + \Sigma_d(6,12,13)$, is:

(a) $\bar{B}\bar{D} + B\bar{C}$ (b) $\bar{B}\bar{D} + B\bar{C} + \bar{A}\bar{B}$ (c) $\bar{D} + B\bar{C}$ (d) $\bar{C}\bar{D} + B\bar{C} + \bar{B}C\bar{D}$

6. The output for the following logic circuit with the given inputs is:




| A | B | Y |
|---|---|---|
| 0 | 1 | Y1 |
| 1 | 1 | Y2 |

(a) $Y1 = 0; Y2 = 0$

(b) $Y1 = 0; Y2 = 1$

(c) $Y1 = 1; Y2 = 0$

(d) $Y1 = 1; Y2 = 1$

7. The resolution of an $n$-bit D to A converter (DAC) using resistive ladder network is:
(a) $1/2^n$ (b) $1/2^{n-1}$ (c) $1/2^n - 1$ (d) $1/2^{n+1}$

8. In a 4-bit serial-in-parallel-out shift register, the outputs $Q_D$, $Q_C$, $Q_B$ and $Q_A$ of respective D Flip-Flops are initially in the state 1010. A binary sequence 0101 is applied to the input of the first D-flip-flop (having output $Q_D$). After two clock pulses, the values of output $Q_D$ $Q_C$ $Q_B$ $Q_A$ of the shift register will be:
(a) 1001 (b) 0100 (c) 0110 (d) 1010

9. The T-states required for the opcode fetch in a 8085 microprocessor and the execution time required for opcode fetch if the clock frequency is 2MHz are:
(a) 4T, 2 μs (b) 8T, 4 μs (c) 2T, 2 μs (d) 8T, 2 μs

10. Assume the accumulator content in 8085 microprocessor is AAH and flag CY = 0. The content of the accumulator and CY flag after the execution of RAL instruction twice are respectively:
(a) A = A9H, CY = 0 (b) A = 54H, CY = 1
(c) A = A9H, CY = 1 (d) A = 54H, CY = 0

11. Which statement will you add in the following program to make it work correctly?

```
#include <stdio.h>
int main () {
printf("%f\n", log (36.0));
    return 0;
    }
```

(a) #include <conio.h> (b) #include <math.h>
(c) #include <stdlib.h> (d) #include <dos.h>

12. Each fusion reaction in the Sun liberates about 26.7 MeV of energy. About $1.4 \times 10^3$W of solar power is incident on each square metre of the Earth that is at a distance of about $1.5 \times 10^{11}$ m from the Sun. The number of fusion reactions occurring per second in the core of the Sun is about:
(a) $10^{37}$ (b) $10^{39}$ (c) $10^{41}$ (d) $10^{43}$

13. A nucleus consists of Z protons and N neutrons. If the mass of proton is $m_p$ and the mass of neutron is $m_n$, the mass of the nucleus, M, is given as:
(a) $M = Zm_p + Nm_n$ (b) $M < Zm_p + Nm_n$
(c) $M > Zm_p + Nm_n$ (d) $M$ is independent of $m_p$ and $m_n$

14. Consider the following reactions:
(I) $p + p \rightarrow p + p + p + \bar{p}$ (II) $p + p \rightarrow p + n + \pi^+$
(III) $p + p \rightarrow p + n + \pi^0 + \pi^+$ (IV) $p + p \rightarrow p + p + \pi^- + \pi^+$
Which of these reactions can occur?
(a) Only I (b) Only IV
(c) Both I and IV (d) All I, II, III and IV

15. In the absence of the neutrino emission, which conservation laws will be violated in beta decay?
(a) Energy conservation only
(b) Energy and momentum conservation only
(c) Energy, momentum and angular momentum conservation
(d) No conservation law is violated

CAREER ENDEAVOUR

16. A mixture contains 2 mole of He ($C_p = 2R, C_v = 1R$) and 1 mole of H ($C_p = 3R, C_v = 2R$). The approximate values of $C_v$ and $\gamma$ for the mixture are, respectively:
(a) $1.33R, 1.75$ (b) $2R, 1.5$ (c) $1.63R, 1.67$ (d) $2.5R, 1.4$

17. A steam engine takes 50g of steam at 100°C per minute and cools it down to 20°C. The heat rejected by the steam engine per minute is about (Latent heat of vaporization of steam = 540 cal/g):
(a) $0.4 \times 10^4$ cal (b) $2.1 \times 10^4$ cal (c) $3.1 \times 10^4$ cal (d) $6.2 \times 10^4$ cal

18. Choose the wrong statement from the following:
(a) Entropy remains constant in all adiabatic processes
(b) Gibbs function increases in all natural processes
(c) Change in enthalpy during an isobaric process is equal to the heat transferred
(d) In a throttling process, the initial and final enthalpies are equal

19. In an adiabatic process involving an ideal gas, if the volume $V$ is related to the absolute temperature $T$, by $V \sim 1/T^2$, the adiabatic index $\gamma (= C_p / C_v)$ is:
(a) 1.4 (b) 1.5 (c) 0.5 (d) 0.75

20. In superconductors, Cooper pairs are formed due to electron-electron interactions mediated by:
(a) Phonons (b) Photons (c) Excitons (d) Polarons

21. Dielectric polarizability is a proportionality constant in the relation between:
(a) Electric dipole moment and local electric field
(b) Displacement vector and local electric field
(c) Electric dipole moment and charge density
(d) Displacement vector and charge density

22. If the molecular weight and density of rock salt (NaCl) with fcc structure are 60 and 2160 kg/m$^3$, respectively, the lattice constant in Å is about:
(a) 6.78 (b) 5.69 (c) 4.51 (d) 3.10

23. Debye temperatures of copper and gold are 340K and 170K, respectively. At room temperature, if the specific heat for copper is 1.5 kJmol$^{-1}$K$^{-1}$, the specific heat for gold is about:
(a) 0.19 kJmol$^{-1}$K$^{-1}$ (b) 1.2 kJmol$^{-1}$K$^{-1}$ (c) 1.9 kJmol$^{-1}$K$^{-1}$ (d) 12 kJmol$^{-1}$K$^{-1}$

24. If a charged particle of charge $q$ and mass $m$ moving with speed $v$ enters a magnetic field $\boldsymbol{B}$ perpendicular to its direction of motion, it will move in a circular orbit of radius $R$. Then,
(a) $R$ is directly proportional to $q/m$ and $|\boldsymbol{B}|$
(b) $R$ is inversely proportional to $q/m$ and $|\boldsymbol{B}|$
(c) $R$ is directly proportional to $q/m$ and inversely proportional to $|\boldsymbol{B}|$
(d) $R$ is inversely proportional to $q/m$ and directly proportional to $|\boldsymbol{B}|$

25. A current I flows in a cylindrical wire of length L and radius $a$, placed in a uniform electric field, E = VL, parallel to the wire (V is the potential difference between the two ends of the wire). Which of the following statements is correct?
(a) The Poynting vector is zero for this system
(b) The magnitude of Poynting vector is $\frac{VI}{2\pi aL}$ and it is directed parallel to the wire
(c) The magnitude of Poynting vector is $\frac{VI}{2\pi aL}$ and it points radially inward
(d) The magnitude of Poynting vector is $\frac{VI}{2\pi aL}$ and it points radially outward

26. Sunlight is incident on the surface of the sea at an angle $\theta$. A sailor viewing through a rotating polaroid can almost block the reflected light by adjusting the angle of the polaroid. The value of $\theta$ is about (Refractive index of water is 1.33):

(a) 57° (b) 53° (c) 49° (d) 41°

27. The skin depth of the electromagnetic waves in a poor conductor is:

(a) $\omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2}$ (b) $\omega\sqrt{\frac{\varepsilon\mu}{2}}$

(c) $\frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$ (d) $\frac{\lambda}{2\pi}$

28. The ensemble average of the product of two functions $f$ and $g$ is equal to:

(a) Product of the ensemble average of $f$ and the ensemble average of $g$
(b) Sum of the ensemble average of $f$ and the ensemble average of $g$
(c) Difference between the ensemble average of $f$ and the ensemble average of $g$
(d) Ratio of the ensemble average of $f$ and the ensemble average of $g$

29. The wave-function for identical fermions is anti-symmetric under interchange of particle positions. This is a consequence of:

(a) Dirac quantization condition
(b) Bose-Einstein condensation
(c) Bohr correspondence principle
(d) Pauli exclusion principle

30. Consider a Maxwell-Boltzmann system with two states of energies $E$ and $2E$. If each state is two-fold degenerate, then the partition function would be (in terms of $E$, $k$, and $T$, where $k$ is the Boltzmann constant and $T$ is the absolute temperature):

(a) $Z = 2e^{-3E/(kT)}$ (b) $Z = e^{-E/(kT)}[1+2e^{-E/(kT)}]$

(c) $Z = 2e^{-E/(kT)}[1+e^{-2E/(kT)}]$ (d) $Z = 2e^{-E/(kT)}[1+e^{-E/(kT)}]$

CAREER ENDEAVOUR

31. The surface of the Sun has a temperature close to 6000 K and it emits a blackbody (Planck) spectrum that reaches a maximum near a wavelength of $5 \times 10^{-7}$ m. For a body with a surface temperature of 300K, the thermal spectrum would reach a maximum at a wavelength:

(a) $10^{-5}$ m (b) $2 \times 10^{-6}$ m (c) $5 \times 10^{-6}$ m (d) $10^{-6}$ m

32. If the monochromatic source of light in Young's double-slit experiment is replaced by a white-light source, then:

(a) No fringes will be formed
(b) There will be white fringes only
(c) There will be a coloured central fringe only
(d) There will be a central white fringe flanked on either side by a few coloured fringes

33. Interference and diffraction of light supports its:

(a) Wave nature (b) Quantum nature
(c) Transverse nature (d) Electromagnetic character

34. Longitudinal waves will not show:
(a) Interference
(b) Diffraction
(c) Polarisation
(d) Total internal reflection

35. The Fourier transform of $\sin(\omega_0 t)$ is proportional to:
(a) $\delta(\omega+\omega_0)$
(b) $\delta(\omega-\omega_0)$
(c) $\delta(\omega+\omega_0)-\delta(\omega-\omega_0)$
(d) $\delta(\omega+\omega_0)+\delta(\omega-\omega_0)$

36. The Fourier transform of a function $f(t)$ is denoted by $\tilde{F}(\omega)$. Let the complex conjugate of $\tilde{F}(\omega)$ be denoted by $\tilde{F}^{'*'}(\omega)$. If $f(t)$ is real, then:
(a) $\tilde{F}(\omega)=\tilde{F}^{'*'}(-\omega)$
(b) $\tilde{F}(\omega)=\tilde{F}^{'*'}(\omega)$
(c) $\tilde{F}(\omega)=\tilde{F}^{'*'}\left(\frac{1}{\omega}\right)$
(d) $\tilde{F}(\omega)=\tilde{F}^{'*'}\left(\frac{-1}{\omega}\right)$

37. A periodic function can be expressed in the form

$$f(x)=\sum_{n=0}^{\infty}\left[a_{2n}\cos nx+a_{2n+1}\sin nx\right]$$

If this function is given to be $f(x)=|\sin(x)|$, which of the following statement is correct?
(a) $a_{2n}=0$ for all $n$
(b) $a_{2n+1}=0$ for all $n$
(c) $a_n=0$ for all $n$
(d) $a_n=0$ for all n other than $n=1$

38. The first and second order Legendre polynomials are given by

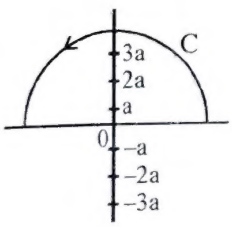
$$P_1(\mu)=\cos(\theta),\ P_2(\mu)=\frac{3}{2}\cos^2(\theta)-\frac{1}{2},\ \text{where } \mu=\cos(\theta).$$

The integral $\int P_1(\mu)P_2(\mu)\sin(\theta)d\theta$ with limits of integral from $\theta=0$ to $\theta=\pi$ will be:
(a) $3\pi$
(b) $-\pi$
(c) 0
(d) $\pi$

39. If an electron of mass $m_e$ and a proton of mass $m_p$ have the same kinetic energy, then the ratio of their de Broglie wavelengths $\lambda_e/\lambda_p$ is:
(a) 1
(b) $m_p/m_e$
(c) $(m_p/m_e)^{1/2}$
(d) $(m_p/m_e)^2$

40. The value of the integral $2\int x^4 e^{-x^2}dx$ with limits $-\infty$ to $+\infty$ is:
(a) $\Gamma(5/2)$
(b) $\Gamma(5)$
(c) $\Gamma(4)$
(d) $\Gamma(3/2)$

41. Consider the integral of the function f(z) in the complex plane along the contour C as shown in the figure. For $f=[z^2+a^2]^{-1}$, the value of the integral is:




(a) $\frac{\pi}{a}$
(b) $\frac{2\pi}{a}$
(c) $\frac{i\pi}{2a}$
(d) $\frac{i\pi}{a}$

CAREER ENDEAVOUR

42. The function $f(z)=\dfrac{(z-2)}{(z^2-4)(z+3)}$

(a) Has a simple pole at z = 2 and a simple pole at z = 3
(b) Has a simple pole at z = –2 and a simple pole at z = –3
(c) Has a pole of order 2 at z = 2 and a simple pole at z = –3
(d) Has a pole of order 2 at z = –2 and a simple pole at z = 3

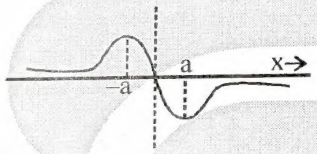
43. The complex function $f(z) = z^{1/2}$

(a) Has a simple pole at z = 0
(b) Has a pole of order 2 at z = 0
(c) Has a branch point
(d) Is analytic in the entire complex plane

44. A particle moves along a circle of unit radius with uniform angular velocity ω. The probability of finding the particle at a position between x and x + dx(x < 1) to the right of the centre, is:

(a) $\dfrac{dx}{\pi\sqrt{1-x^2}}$ (b) $\dfrac{dx(1-x^2)}{2\pi}$ (c) $\dfrac{dx}{2\pi(1-x^2)}$ (d) $\dfrac{dx\sqrt{1-x^2}}{2\pi}$

45. The wave function of the first excited state of a particle in a simple harmonic oscillator potential may be represented as shown in the figure below:




The probability of finding the particle at a particular position is maximum for:
(a) only x = a
(b) only x = –a
(c) Both x = a and x = –a
(d) x tending to $\pm\infty$

46. Which of the following is an admissible wave function in three dimensions? (Here $N_1$, $N_2$, $N_3$, $N_4$ and $r_0$ are constants and r is the radial coordinate):

(a) $\psi=\dfrac{N_1}{(r^2+r_0^2)^{1/4}}$ (b) $\psi=\dfrac{N_2}{(r^2+r_0^2)^{1/2}}$

(c) $\psi=\dfrac{N_3}{(r^2+r_0^2)^{3/2}}$ (d) $\psi=\dfrac{N_4}{(r^2+r_0^2)^2}$

47. The wave function of a particle is given by $\psi=\dfrac{1}{\sqrt{2}}e^{-|x|/2}$ with $-\infty<x<+\infty$. The minimum uncertainty in the momentum is:

(a) $10h/(2\pi)$ (b) $h/(2\pi\sqrt{2})$ (c) $2/h$ (d) $\sqrt{h}$

48. The general solution of the differential equation $\dfrac{d^3y}{dx^3}+\dfrac{1}{x}\dfrac{d^2y}{dx^2}+\dfrac{1}{x^2}\dfrac{dy}{dx}=0$ can be expressed as ($C_1$, $C_2$ and $C_3$ are constants):

(a) $C_1 + C_2x^{1+i} + C_3x^{1-i}$
(b) $C_1 + C_2x + C_3x^i$
(c) $C_1x + C_2x^2 + C_3x^3$
(d) $C_1x + C_2x^{1+i} + C_3x^{1-i}$

CAREER ENDEAVOUR

49. A system is described by a Lagrangian $[dx/dt]^3 + x^3$ where x is the degree of freedom and t is the time. The Euler equation of motion for this system is,

(a) $\frac{d^2x}{dt^2} = -3x^2$

(b) $\frac{d^2x}{dt^2} = 3x^2$

(c) $\frac{d^2x}{dt^2} = \frac{1}{2}\frac{x^2}{dx/dt}$

(d) $\frac{d^2x}{dt^2} = -\frac{1}{2}\frac{x^2}{dx/dt}$

50. The most general solution for the one dimensional wave equation $\frac{\partial^2 F}{\partial x^2} = \frac{\partial^2 F}{\partial t^2}$ is:

(a) $F = h(x + t) + g(x - t)$

(b) $F = h(x - t)$

(c) $F = g(x - t)$

(d) $F = h(x + t) \times g(x - t)$

51. Consider a partial differential equation given by $\frac{\partial^2 F}{\partial x^2} = \frac{\partial F}{2\partial t}$. An admissible solution will be:

(a) $e^{kx+2\sqrt{k}t} + e^{kx-2\sqrt{k}t}$

(b) $e^{\sqrt{k}x+2kt} + e^{-\sqrt{k}x+2kt}$

(c) $e^{kx-2\sqrt{k}t}$

(d) $e^{kx+2\sqrt{k}t}$

52. If the trace and the determinant of a $2 \times 2$ matrix are both zero; then,

(a) Only one of eigenvalues needs to be zero

(b) Both the eigenvalues need to be zero

(c) Both the eigenvalues are non-zero but finite

(d) At least one of the eigenvalues should be infinite

53. The matrix given by:

$$\begin{pmatrix} 3 & 2i & 1 \\ -2i & 4 & 1+2i \\ 1 & 1-2i & 5 \end{pmatrix}$$

(a) Has one real and one complex eigenvalues

(b) Has two real and one complex eigenvalues

(c) Has three real eigenvalues

(d) Has three complex eigenvalues

54. The trace of a $4 \times 4$ matrix is 8 and its determinant is $-24$. If two of its eigenvalues are 3 and 4, the other two eigenvalues are:

(a) $-2$ and 1 (b) 2 and $-1$ (c) 2 and 1 (d) $-2$ and $-1$

55. A vector field is given by $\vec{A} = y\hat{i} + z\hat{j} + x\hat{k}$. The curl of the vector is:

(a) $\hat{i} + \hat{j} - \hat{k}$ (b) $-\hat{i} - \hat{j} - \hat{k}$ (c) $-\hat{i} - \hat{j} + \hat{k}$ (d) $\hat{i} + \hat{j} + \hat{k}$

56. The magnitude of the cross product of two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is equal to:

(a) The area of the triangle

(b) The area of the parallelogram with adjacent sides AB and AC

(c) Twice the area of the rectangle with adjacent sides AB and AC

(d) The area of the circle passing through A, B and C

57. Consider a set of surfaces $f(x, y, z) = c$. Different values of $c$ will generate different surfaces. The gradient of $f$ at a point P has a direction:

(a) Tangential to the surface at point P

(b) At 45 degrees to the surface at point P

(c) Normal to the surface at the point P in the direction of increasing value of $f$

(d) Normal to the surface at the point P in the direction of decreasing value of $f$

58. A wire is in the form of a curve given by $2y = x^2$ with $-1 < x < 1$. This is rotated by an angle $\pi$ about the line $x = 0$. The area swept by the wire is:

(a) $\frac{2\pi}{3}$ (b) $(2^{3/2})\frac{2\pi}{3}$ (c) $(2^{3/2}-1)\frac{2\pi}{3}$ (d) $(2^{3/2}+1)\frac{2\pi}{3}$

59. Apply Newton-Raphson method to solve the equation $(1/x) - a = 0$. The resulting iterative formula for the root of this equation is:

(a) $x_{n+1} = 2x_n + ax_n^2$ (b) $x_{n+1} = 2x_n - ax_n^2$ (c) $x_{n+1} = ax_n^2$ (d) $x_{n+1} = -ax_n^2$

60. Which of the following is not an iterative method for solving equations?

(a) Fixed point method (b) Newton - Raphson method
(c) Gauss elimination method (d) Gauss - Seidel method

61. It is known that the root of the function $f(x)$ lies between 0 and 1. After 10 iterations of the bisection method, the approximate accuracy of the root will be:

(a) Correct up to 2 places of decimal (b) Correct up to 3 places of decimal
(c) Correct up to 4 places of decimal (d) Correct up to 5 places of decimal

62. The electric field of a travelling wave is given by $100\cos(10^9 t - 4x)$, where $t$ is in seconds and x in metres. The speed of the wave is:

(a) $3.0 \times 10^8$ m/s (b) $2.5 \times 10^8$ m/s (c) $3.0 \times 10^7$ m/s (d) $5.0 \times 10^7$ m/s

63. Electric susceptibility $\chi$ of a linear, isotropic and non-dispersive medium is related to the electric displacement vector $\boldsymbol{D}$ and the applied electric field $\boldsymbol{E}$ via the relation:

(a) $\chi = |\mathbf{D} - 4\pi\mathbf{E}|$ (b) $\chi = |\mathbf{D} - \mathbf{E}|/4\pi$
(c) $\chi\mathbf{E} = \mathbf{D} - 4\pi\mathbf{E}$ (d) $\chi\mathbf{E} = (\mathbf{D} - \mathbf{E})/4\pi$

64. The Poynting vector has the dimension of:

(a) Energy / Area (b) Power / Area
(c) Energy / (Charge × Volume) (d) Power / (Charge × Volume)

65. The phase difference between two points, which are 0.5 cm apart in a wave of wavelength 1.0 metre, is:

(a) 2.8° (b) 5.0° (c) 3.6° (d) 1.8°

66. The motion of a simple pendulum executing oscillations with large amplitude is:

(a) Periodic and simple harmonic (b) Periodic but not simple harmonic
(c) Aperiodic (d) Rectilinear

67. A vertical spring, loaded with a mass, executes simple harmonic motion with a period of 4s. The difference in the kinetic energy and potential energy of this system changes with a period of:

(a) 4s (b) 2s (c) 8s (d) 16s

68. Consider an air column having one end closed. The ratio of lengths of columns having the other end either open or closed, which emit the same fundamental note, is:

(a) 4:1 (b) 3:2 (c) 8:1 (d) 2:1

69. Splitting of spectral lines in the Stark effect depends on:

(a) Magnetic quantum number (b) Spin quantum number
(c) Azimuthal quantum number (d) Principal quantum number

70. A three-level system of atoms has $N_1$ atoms in level $E_1$, $N_2$ in level $E_2$ and $N_3$ in level $E_3$ ($N_2 > N_1 > N_3$ and $E_1 < E_2 < E_3$). Laser emission is possible for the transitions:

(a) $E_2 \rightarrow E_1$ (b) $E_3 \rightarrow E_2$ (c) $E_2 \rightarrow E_3$ (d) $E_3 \rightarrow E_1$

71. The ratio of intensities of spectral lines in hydrogen spectrum for the transitions $2^2p_{1/2} \rightarrow 1^2s_{1/2}$ is:

(a) 1/4 (b) 1/6 (c) 1/3 (d) 1/2

72. A satellite of mass $m$ is moving round a planet of mass $M$ in a circular orbit of radius $R$. The time taken for one complete revolution is:
(a) Linear in $R$
(b) Proportional to $m^{1/2}$
(c) Proportional to $R^{3/2}$
(d) Independent of $M$

73. If the sum of all external forces on a system of interacting particles is zero, then:
(a) The total mechanical energy of the system is necessarily constant
(b) The total potential energy of the system is necessarily constant
(c) The total linear momentum of the system is necessarily constant
(d) The total kinetic energy of the system is necessarily constant

74. A horizontally mounted disk with moment of inertia $I$ spins about a frictionless axle. At time $t = 0$, the initial angular speed of the disk is $\omega$. A constant torque $\tau$ is applied to the disk, causing it to come to a halt in time $t$. The power required to dissipate the wheel's energy during this time is:
(a) $(I\omega^2)/(2t)$ (b) $(I\omega^2)/(2\tau)$ (c) $(I\omega)/t$ (d) $(I\omega^2)/2$

75. Relative to a stationary observer, a rod of length 1.0 metre is moving at 0.8 times the speed of light in vacuum. It would appear to the observer that the rod's length is:
(a) 0.8 m (b) 0.6 m (c) 1.0 m (d) 1.25 m

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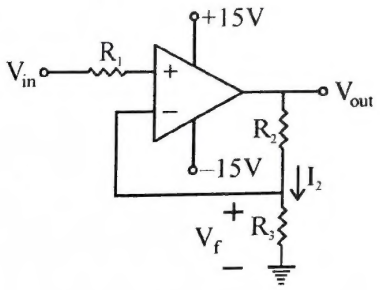
CAREER ENDEAVOUR

# SOLVED PAPER : DU 2014

## PHYSICS-PH

## OBJECTIVE QUESTIONS

1. It is a voltage series negative feedback amplifier.




$$\therefore \quad R_{if} = R_i\left(1 + A_{OL}\beta\right)$$

$$= 2M\Omega\left(1 + 2\times10^5 \times \frac{1}{40}\right)$$

$$V_f = \frac{V_{out} \times R_3}{R_3 + R_2} \Rightarrow \frac{V_f}{V_{out}} = \beta = \frac{1}{40}$$

$$= 10\ G\Omega$$

$$\& \quad R_{0f} = \frac{R_0}{1 + A_{OL}\beta} = \frac{75}{1 + 2\times10^5 \times \frac{1}{40}}$$

$$R_{0f} = 0.015\ \Omega$$

**Correct option is (b)**

2. The efficiency of class-B push-pull configuration is given by

$$\eta = \frac{\pi}{4}\left(1 - \frac{V_{min}}{V_{CC}}\right)$$

For maximum efficiency $V_{min} \ll V_{CC}$

Therefore, $\eta_{max} = \frac{\pi}{4}$

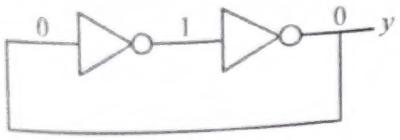
**Correct option is (b)**

3. $$g_m = g_{m_0}\left[1 - \frac{V_{GS}}{V_P}\right] \Rightarrow g_{m_0} = \left|-\frac{2I_{DSS}}{V_P}\right|$$

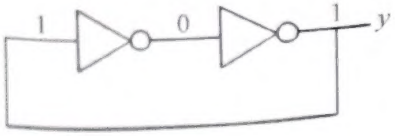
$$1 = g_{m_0}\left[1 - \frac{-3}{-5}\right] \Rightarrow g_{m_0} = 2.5\,\frac{mA}{V}$$

$g_{m_0}$ = maximum transconductance = 2.5 mA/V

**Correct option is (c)**

4. 


$y = 0$




$y = 1$

$y$ = either 0 OR 1

**Correct option is (b)**

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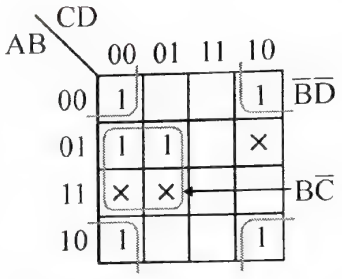
5. $Y = \Sigma_m(0, 2, 4, 5, 8, 10) + \underbrace{\Sigma_d}_{\text{don't care}}(6, 12, 13)$ $\underbrace{1101}_{\text{4 variable K-map}}$

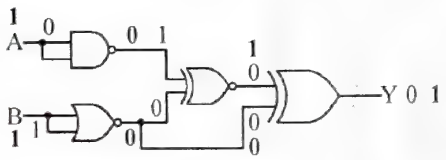



$Y = \bar{B}\bar{D} + B\bar{C}$

**Correct option is (a)**

6. Given bold case - (2)

Given non-bold case - (1)




**Correct option is (b)**

7. $\text{Resolution} = \dfrac{1}{2^n - 1}$

**Correct option is (c)**

8.

| CLK | Input | $Q_D$ | $Q_C$ | $Q_B$ | $Q_A$ |
|---|---|---|---|---|---|
| | | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 2 | 1 | 1 | 0 | 1 | 0 |
| | 0 | | | | |
| | 1 | | | | |

So after 2nd clock output will be 1010.

**Correct option is (d)**

9. 99 % of opcode fetch for any instruction in microprocessor 8085 has '4T' opcode fetch. '6T' for **"Return with condition"**

RZ Return zero
RNZ Return not zero
RC Return carry
RNC Return no carry
RPO Return parity odd
RPE Return parity even
RP Return positive

RM Return minus
'6T' for PUSH
SPHL
INX

Execution time = $4T = \frac{4}{f} \leftarrow 2 \times 10^6$

$t = 2\,\mu sec$

**Correct option is (a)**

10. RAL : It is for rotate the accumulator left through carry.
Initially :

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A = | A | A | H | | | | | | CY = 0 |
| A = | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | CY = 0 |
| | $D_7$ | $D_6$ | $D_5$ | $D_4$ | $D_3$ | $D_2$ | $D_1$ | $D_0$ | |

RAL first time applied then

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | CY ↓ | |
| A = | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | CY = 1 |
| | $D_7$ | $D_6$ | $D_5$ | $D_4$ | $D_3$ | $D_2$ | $D_1$ | $D_0$ | ↑ |
| | | | | | | | | | $D_7$ |

RAL second time applied then

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A = | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | CY = 0 |
| | | | | | | | | ↑ | ↑ |
| | | | | | | | | CY | $D_7$ |

A = A9H and CY = 0

**Correct option is (a)**

11. #include <math.h> statement will be added in the following program to make it work correctly, because in program we have log( ) function.

**Correct option is (b)**

12. Total energy released by the source

$E_t = 4\pi R^2 \times$ power per unit area

$= 4\pi (1.5 \times 10^{11})^2 \times 1.4 \times 10^3$ W

Energy released per fusion $E_1 = 26.7$ MeV $= 26.7 \times 10^6 \times 1.6 \times 10^{-19}$ J

Total number of fusion per second

$$N = \frac{E_t}{E_1} = \frac{4\pi \times (1.5 \times 10^{11})^2 \times 1.4 \times 10^3}{26.7 \times 10^6 \times 1.6 \times 10^{-19}\ \text{J}}$$

$$= \frac{4\pi \times (1.5)^2 \times 1.4}{26.7 \times 1.6} \times 10^{22+3-6+19} = 0.92 \times 10^{38}$$

$$= 9.2 \times 10^{37}$$

**Correct option is (a)**

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13. Mass of the nucleus is always less than the total mass of its constituents particles. Some mass energy of the constituents particle is used up as a binding energy of the nucleus.

Therefore, $M < Zm_p + Nm_n$

**Correct option is (b)**

14. (I) $p + p \longrightarrow p + p + p + \bar{p}$

Charge [Q] : $(+1)+(+1) \longrightarrow (+1)+(+1)+(+1)+(-1) \Rightarrow \Delta Q = 0$, charge is conserved

Baryon No. (B) : $(+1)+(+1) \longrightarrow (+1)+(+1)+(+1)+(-1) \Rightarrow \Delta B = 0$, baryon no. is conserved

Strangeness [S]: $(0)+(0) \longrightarrow (0)+(0)+(0)+(0) \Rightarrow \Delta S = 0$, strangeness is conserved

Isospin [I]: $\left(\frac{1}{2}\right) \oplus \left(\frac{1}{2}\right) \longrightarrow \left(\frac{1}{2}\right) \oplus \left(\frac{1}{2}\right) \oplus \left(\frac{1}{2}\right) \oplus \left(\frac{1}{2}\right) \Rightarrow \Delta I = 0$, isospin is conserved.

where, $\oplus$ represents vector law of addition.

Third component of Isospin [$I_3$]:

$$\left(+\frac{1}{2}\right)+\left(+\frac{1}{2}\right) \longrightarrow \left(+\frac{1}{2}\right)+\left(+\frac{1}{2}\right)+\left(+\frac{1}{2}\right)+\left(-\frac{1}{2}\right) \Rightarrow \Delta I_3 = 0,$$

third component of Isospin is conserved.

Hence, this reaction is allowed via strong interaction.

(II) $p + p \longrightarrow p + n + \pi^+$

Charge [Q] : $(+1)+(+1) \longrightarrow (+1)+(0)+(+1) \Rightarrow \Delta Q = 0$

Baryon No. (B) : $(+1)+(+1) \longrightarrow (+1)+(+1)+(0) \Rightarrow \Delta B = 0$

Strangeness [S]: $(0)+(0) \longrightarrow (0)+(0)+(0) \Rightarrow \Delta S = 0$

Isospin [I]: $\left(\frac{1}{2}\right) \oplus \left(\frac{1}{2}\right) \longrightarrow \frac{1}{2} \oplus \frac{1}{2} \oplus 1 \Rightarrow \Delta I = 0$

where, $\oplus$ represents vector law of addition.

Third component of Isospin [$I_3$]: $\left(+\frac{1}{2}\right)+\left(+\frac{1}{2}\right) \longrightarrow \left(+\frac{1}{2}\right)+\left(-\frac{1}{2}\right)+(+1) \Rightarrow \Delta I_3 = 0$

Hence, this reaction is allowed via strong interaction.

(III) $p + p \longrightarrow p + n + \pi^0 + \pi^+$

Charge [Q] : $(+1)+(+1) \longrightarrow (+1)+(0)+(0)+(+1) \Rightarrow \Delta Q = 0$

Baryon No. (B) : $(+1)+(+1) \longrightarrow (+1)+(+1)+(0)+(0) \Rightarrow \Delta B = 0$

Strangeness [S]: $(0)+(0) \longrightarrow (0)+(0)+(0)+(0) \Rightarrow \Delta S = 0$

Isospin [I]: $\left(\frac{1}{2}\right) \oplus \left(\frac{1}{2}\right) \longrightarrow \left(\frac{1}{2}\right) \oplus \left(\frac{1}{2}\right) \oplus (1) \oplus (1) \Rightarrow \Delta I = 0$

where, $\oplus$ represents vector law of addition.

Third component of Isospin [$I_3$]:

$$\left(+\frac{1}{2}\right)+\left(+\frac{1}{2}\right)\longrightarrow\left(+\frac{1}{2}\right)+\left(-\frac{1}{2}\right)+(0)+(+1)\Rightarrow \Delta I_3=0$$

Hence, this reaction is allowed via strong interaction.

(IV) $p+p\longrightarrow p+n+\pi^-+\pi^+$

Charge [Q] : $(+1)+(+1)\longrightarrow(+1)+(+1)+(-1)+(+1)\Rightarrow \Delta Q=0$

Baryon No. (B) : $(+1)+(+1)\longrightarrow(+1)+(+1)+(0)+(0)\Rightarrow \Delta B=0$

Strangeness [S]: $(0)+(0)\longrightarrow(0)+(0)+(0)+(0)\Rightarrow \Delta S=0$

Isospin [I]: $\left(\frac{1}{2}\right)\oplus\left(\frac{1}{2}\right)\longrightarrow\left(\frac{1}{2}\right)\oplus\left(\frac{1}{2}\right)\oplus(1)\oplus(1)\Rightarrow \Delta I=0$

where, $\oplus$ represents vector law of addition.

Third component of Isospin [$I_3$]:

$$\left(+\frac{1}{2}\right)+\left(+\frac{1}{2}\right)\longrightarrow\left(+\frac{1}{2}\right)+\left(+\frac{1}{2}\right)+(-1)+(+1)\Rightarrow \Delta I_3=0$$

Hence, this reaction is allowed via strong interaction.

Thus, from the above discussion, we see that all the four nuclear reaction involving elementary particles are allowed via strong interaction.

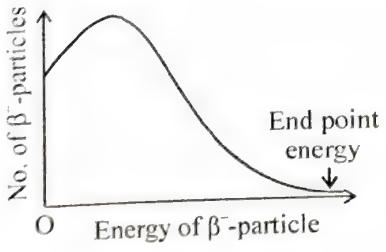
**Correct option is (d)**

15. The early experiments showed that in $\beta^-$-decay a nucleus decays to another nucleus of the same mass number with the emission of an electron with a varying amount of energy. If just an electron was emitted in the $\beta^-$-decay, then energy and momentum (linear and angular) conservation would require all the electrons to have the same energy determined by $E_0$, the $Q$-value for the decay which is equal to the difference in rest mass energies of the initial and final state. The shape of $\beta^-$-spectrum [shown in the figure] implies that the emitted electrons have kinetic energies in a range. Thus, it is clear that another particle is also emitted with the electron. Pauli have postulated neutrino hypothesis. According to Pauli,

(i) neutrinos have zero charge

(ii) neutrinos have zero or almost zero mass.

(iii) They have spin of ½




Thus, $\beta^-$-decay process corresponds to

$$\underset{(neutrino)}{n}\longrightarrow \underset{(proton)}{p}+\underset{(electron)}{e^-}+\underset{(anti-neutrino)}{\bar{\nu}}$$

Spin (s): $\frac{1}{2}\longrightarrow\frac{1}{2}\oplus\frac{1}{2}\oplus\frac{1}{2}\Rightarrow \Delta S=0$ i.e. spin angular momentum is conserved

Thus, from above discussion, it is clear that in the absence neutrino emission, energy, momentum and angular momentum conservation are violated.

**Correct option is (c)**

16. $(C_V)_{\text{mix}} = \dfrac{n_1 (C_V)_1 + n_2 (C_V)_2}{n_1 + n_2} = \dfrac{2 \times 1R + 1 \times 2R}{2+1}$

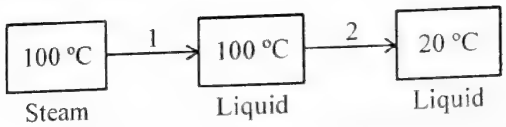
$= \dfrac{4}{3} R \simeq 1.33\, R$

$\therefore \quad (C_P)_{\text{mix}} = (C_V)_{\text{mix}} + R = \dfrac{7}{3} R$

$\therefore \quad (\gamma)_{\text{mix}} = \dfrac{(C_P)_{\text{mix}}}{(C_V)_{\text{mix}}} = \dfrac{7}{4} = 1.75$

**Correct option is (a)**

17.




$\therefore \quad \Delta Q = \Delta Q_1 + \Delta Q_2$

$= -50 \times 540 + 50 \times 1 \times (20 - 100)$

$= -3.1 \times 10^4$ cal

**Correct option is (c)**

18. In all natural processes, thermodynamic potentials gets minimized so as to attain equilibrium by the system. Also, entropy does not remain constant in all adiabatic processes. Rather entropy of an ioslated system increases in reversible adiabatic process.

**Correct options are (a) & (b)**

19. In adiabatic process, we have

$TV^{\gamma-1} = \text{constant}$

$\Rightarrow V = \left( \dfrac{\text{constant}}{T} \right)^{1/(\gamma-1)}$

$\therefore \quad \dfrac{1}{\gamma - 1} = 2 \Rightarrow \gamma = 1.5$

**Correct option is (b)**

20. Cooper pairs are formed due to electron-electron interaction mediated by phonons.

**Correct option is (a)**

21. Dielectric polarizability $= \dfrac{\text{induced dipole}}{\text{local electric field}}$

$\alpha = \dfrac{\bar{p}}{\bar{E}}$

**Correct option is (a)**

22. NaCl has FCC structure. So, total number molecule in unit cell is $n = 4$.
Therefore, the density of NaCl is

$\rho = \dfrac{mn}{V} = \dfrac{M}{N_A} \dfrac{n}{a^3}$

$$\Rightarrow a^3 = \left(\frac{Mn}{N_A \rho}\right) = \frac{60 \times 4}{6.023 \times 10^{26} \times 2160} m^3$$

$$= 0.0184 \times 10^{-26}\ m^3$$

$$\therefore\ a = (184 \times 10^{-30}\ m^3)^{1/3} = 5.69\ \text{Å}$$

**Correct option is (b)**

23. According to Debye theory the specific heat is given by

$$C_V = \frac{12}{5}\pi^4 N k_B \left(\frac{T}{\theta_D}\right)^3$$

$$\therefore\ \frac{(C_V)_{Cu}}{(C_V)_{Au}} = \left(\frac{\theta_{DAu}}{\theta_{DCu}}\right)^3$$

$$\Rightarrow (C_V)_{Au} = (C_V)_{Cu}\left(\frac{\theta_{DCu}}{\theta_{DAu}}\right)^3$$

$$= 1.5 \times \left(\frac{340}{170}\right)^3 = 1.5 \times 8 = 12\ \text{kJ mol}^{-1}\ \text{K}^{-1}$$

**Correct option is (d)**

24. The magnetic force act on the charge particle is given by

$$F = q\bar{v} \times \bar{B} = qvB\sin 90° = qvB$$

For circular motion,
Centrifugal force must be equal to centripetal force

$$\therefore\ \frac{mv^2}{R} = qvB$$

$$\Rightarrow\ R = \left(\frac{mv}{qB}\right)$$

So $R \propto \left(\frac{m}{q}\right)$ and $R \propto \frac{1}{B}$

**Correct option is (d)**

25. Suppose electric field $\vec{E} = V / L\,\hat{z}$
So current will flow along z-axis
So magnetic field on the surface of the cylinder

$$\oint \bar{B}.\overline{dl} = \mu_0 I \ \Rightarrow\ B = \left(\frac{\mu_0 I}{2\pi a}\right) \Rightarrow \qquad \vec{H} = \frac{I}{2\pi a}\hat{\phi}$$

The Poynting vector is given by

$$\vec{S} = \vec{E} \times \vec{H} = \frac{VI}{2\pi aL}(\hat{z} \times \hat{\phi}) = \frac{VI}{2\pi aL}(-\hat{r})$$

Therefore, the magnitude of Pyonting vector = $\frac{VI}{2\pi aL}$ and it points radially inward.

**Correct option is (c)**

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26. Only linear polarized light can be blocked using polaroid, so the reflected light must be linear polarized. When light incident at Brewster angle, then the reflected light becomes linear polarized

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{1.33}{1} \Rightarrow \theta_p = \tan^{-1} 1.33 = 53^\circ$$

**Correct option is (b)**

27. Skin depth is given by

$$d = \frac{1}{k_i}, \text{ where } k_i = \omega\sqrt{\frac{\varepsilon u}{2}}\left[\left(1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2\right)^{1/2} - 1\right]^{1/2}$$

For poor conductor $\sigma << \varepsilon\omega$

$$\therefore \quad k_i = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[1+\frac{1}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^2 - 1\right]^{1/2}$$

$$= \omega\sqrt{\frac{\varepsilon\mu}{4}}\frac{\sigma}{\varepsilon\omega} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$\therefore$ The skin depth $d = \frac{1}{k_i} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$

**Correct option is (c)**

28. For two independent function $f$ and $g$, we have

$$\langle fg\rangle = \langle f\rangle\langle g\rangle$$

**Correct option is (a)**

29. The composite wave function for two fermions $a$ and $b$ is given by

$$\psi(\vec{r}_1, \vec{r}_2) = A\left[\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) - \psi_b(\vec{r}_1)\psi_a(\vec{r}_2)\right]$$

If both fermions are in same state i.e. $a = b$, then

$$\psi(\vec{r}_1, \vec{r}_2) = A\left[\psi_a(\vec{r}_1)\psi_a(\vec{r}_2) - \psi_a(\vec{r}_1)\psi_a(\vec{r}_2)\right] = 0$$

Therefore, it means that two fermions cannot occupy the same state, which is known as Pauli's exclusion principle.

**Correct option is (d)**

30. $$z = \sum_i g_i e^{-E_i/kT}$$

$$= 2e^{-E/kT} + 2e^{-2E/kT}$$

$$= 2e^{-E/kT}\left[1 + e^{-E/kT}\right]$$

**Correct option is (d)**

31. According to Wien's displacement law, $\lambda_m T = \text{constant}$, where $\lambda_m$ is wavelength at which intensity of the Blackbody radiation attains maximum and $T$ is absolute temparature of the blackbody.

According to the question, $5\times10^{-7}\times6000 = \lambda_m \times 300 \Rightarrow \lambda_m = 10^{-5}$ m

**Correct option is (a)**

32. All the components of white light have path difference zero at the central point of screen. Therefore, central fringe will be white corresponding of $n = 0$.
**Correct option is (d)**

33. Interference and diffraction phenomena of light supports the wave nature of light.
**Correct option is (a)**

34. In longitudinal wave particles vibrate in the direction of wave propagation. Therefore, longitudinal waves can't be polarized.
**Correct option is (c)**

35. $f(t) = \sin(\omega_0 t)$

The Fourier transform of $f(t)$ is $\tilde{f}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t)\,dt$

$$\therefore \quad \tilde{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} \sin(\omega_0 t)\,dt \quad \Rightarrow \quad \tilde{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t}\left(\frac{e^{i\omega_0 t} - e^{-i\omega_0 t}}{2i}\right)dt$$

$$\Rightarrow \quad \tilde{f}(\omega) = \frac{1}{2i}\left[\int_{-\infty}^{\infty} e^{i(\omega_0 - \omega)t}\,dt - \int_{-\infty}^{\infty} e^{-i(\omega + \omega_0)t}\,dt\right]$$

$$\Rightarrow \quad \tilde{f}(\omega) = \frac{\pi}{i}[\delta(\omega_0 - \omega) - \delta(\omega_0 + \omega)]\left(\because \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}dx = \delta(k)\right)$$

**Correct option is (c)**

36. $\tilde{F}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)\,dt$

Taking complex conjugate of the above equation, we have

$$\tilde{F}^*(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} f(t)\,dt \qquad [f(t) \text{ is real}]$$

$$\Rightarrow \quad \tilde{F}^*(\omega) = \tilde{F}(-\omega)$$

Again taking complex conjugate of the above equation, we have

$$\tilde{F}(\omega) = \tilde{F}^*(-\omega)$$

**Correct option is (a)**

37. $f(x) = \sum_{n=0}^{\infty}[a_{2n}\cos nx + a_{2n+1}\sin nx]$

Since, $f(x) = |\sin x|$ is an even function, there will not be any sine term in the Fourier expansion of $f(x)$.

$\Rightarrow a_{2n+1} = 0$ for all $n$

**Correct option is (b)**

38. Given: Legendre polynoimals $P_1(\mu) = \cos(\theta)$, $P_2(\mu) = \frac{3}{2}\cos^2\theta - \frac{1}{2}$, $\mu = \cos\theta$

We can have $d\mu = -\sin\theta\, d\theta$

$$\therefore \int_0^{\pi} P_1(\mu)\, P_2(\mu) \sin\theta\, d\theta = \int_1^{-1} P_1(\mu)\, P_2(\mu) d\mu$$

According to the orthonormal property of the Legendre polynomials,

$$\Rightarrow \int_{-1}^{1} P_1(\mu) P_2(\mu) d\mu = 0$$

**Correct option is (c)**

39. Kinetic energy $= \frac{1}{2}mv^2$ $\quad [m \equiv \text{mass, and } v \equiv \text{speed}]$

Here, $(\text{K.E})_{\text{proton}} = (\text{K.E})_{\text{electron}}$

$$\therefore \frac{1}{2} m_p v_p^2 = \frac{1}{2} m_e v_e^2 \Rightarrow \frac{v_p}{v_e} = \left(\frac{m_e}{m_p}\right)^{1/2}$$

Now, de-Broglie wavelength $(\lambda) = \frac{h}{p}$, where $p$ is momentum of the particle.

$$\therefore \frac{\lambda_e}{\lambda_p} = \frac{m_p v_p}{m_e v_e} = \frac{m_p}{m_e}\sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{m_p}{m_e}}$$

**Correct option is (c)**

40. Using $\int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx = \Gamma\left(n + \frac{1}{2}\right)$, we have $2\int_{-\infty}^{\infty} x^4 e^{-x^2} dx = 2\Gamma\left(2 + \frac{1}{2}\right) = 2\Gamma\left(\frac{5}{2}\right)$

**None of the options given are correct.**

41. $$\int_C f(z)\, dz = \int_C \frac{dz}{(z^2 + a^2)} = \int_C \frac{dz}{(z + ia)(z - ia)}$$

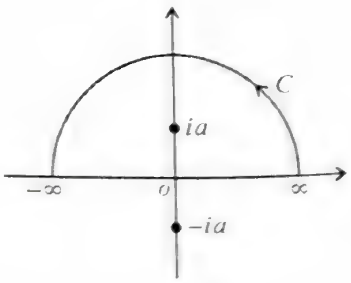
[$f(z)$ has simple poles at $z = ia, -ia$ and $z = -ia$ lies outside the given closed cuve]

$= 2\pi i \times$ Residue of $f(z)$ at $z = ia$

$$= 2\pi i \times \lim_{z \to ia} [(z - ia) f(z)]$$

$$= 2\pi i \times \lim_{z \to ia} \frac{(z - ia)}{(z - ia)(z + ia)}$$

$$= 2\pi i \times \frac{1}{2ia} = \frac{\pi}{a}$$




**Correct option is (a)**

42. $f(z)=\dfrac{(z-2)}{(z^2-4)(z+3)} \quad =\dfrac{(z-2)}{(z-2)(z+2)(z+3)} \quad =\dfrac{1}{(z+2)(z+3)}$

Poles of $f(z)$ are given by $(z+2)(z+3)=0 \Rightarrow z=-2,-3$.

Therefore, $f(z)$ has simple poles at $z=-2,-3$.

**Correct option is (b)**

43. $f(z)=z^{1/2}$ is a multi-valued function.

Here, $f'(z)=\dfrac{1}{2\sqrt{z}}$ is not defined at z = 0. Therefore, $f(z)$ is not differentiable/analytic at z = 0.

So, z = 0 is a singular point of the function $f(z)$. For multi-valued functions, singular points are called branch points

**Correct option is (c)**

44. The probability of finding the particle within $x$ and $x+dx$

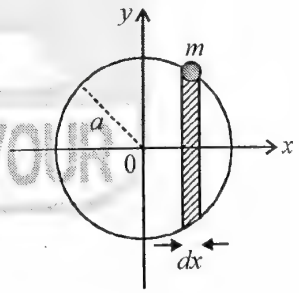
$$=\frac{\text{time spent by the particle in the region within } x \text{ and } x+dx}{\text{time period of oscillation}}$$

$\Rightarrow p(x)\,dx=\dfrac{2dt}{T}$, where $dt$ is the time taken for the particle to cover distance $dx$.

$$=\frac{2dt}{2\pi/\omega}=\frac{\omega dt}{\pi}$$

We have, $x=a\sin(\omega t+\phi)$, [$a$ = amplitude = radius]

$$\Rightarrow dx=a\omega\cos(\omega t+\phi)\,dt \Rightarrow dt=\frac{dx}{a\omega\sqrt{1-\dfrac{x^2}{a^2}}}=\frac{dx}{\omega\sqrt{a^2-x^2}}$$




Given : $a=1$

So, $p(x)\,dx=\dfrac{\omega}{\pi}\dfrac{dx}{\omega\sqrt{1^2-x^2}}=\dfrac{dx}{\pi\sqrt{1-x^2}}$

**Correct option is (a)**

45. The probability of finding a particle within an infinitesimal with $dx$ about a point $x$ is given by $p(x)=|\psi(x)|^2\,dx$.

Here, $\psi(x)$ has a maxima values at $x=-a$ & has a minima $x=a$.

Therefore, $|\psi(x)|^2$ will have maxima at $x=a,-a$.

Since, $p(x)$ depends on $|\psi(x)|^2$. The probability of finding the particle will be highest at both $x=\pm a$.

**Correct option is (c)**

46. For a 3-D function $\psi(\vec{r})$

$$\int_{r=0}^{\infty}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}|\psi(\vec{r})|^2 r^2 \sin\theta\, dr\, d\theta\, d\phi = 1$$

$\therefore$ Here, $\int_{r=0}^{\infty}|\psi(r)|^2 r^2 dr = \text{finite}$

$\therefore$ $|\psi(r)|^2$ should fall faster than $\frac{1}{r^3}$ $\left[\text{Since, } r^2 dr \text{ goes like } r^3\right]$

For $\psi = \frac{N_3}{(r^2 + r_0^2)^{3/2}}$; $\lim_{r \gg r_0} \psi^2 = \frac{N_3^2}{r^3}$

Also, for $\psi = \frac{N_4}{(r^2 + r_0^2)^2}$; $\lim_{r \gg r_0} \psi^2 = \frac{N_4^2}{r^8}$

The other two don't obey it.

Besides this, we know that an accepatble wave function should be single valued in nature. The function given in option (c) is a multivauled function. Hence, it is not acceptable as a wave function. 49.

**Correct option is (d)**

47. $\psi(x) = \frac{1}{\sqrt{2}} e^{-|x|} \quad (-\infty < x < \infty)$

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \psi\, dx = \int_{-\infty}^{\infty} \frac{x}{2} e^{-|x|} dx = 0 \qquad [\because \text{integrand is an odd function}]$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \psi^* x^2 \psi\, dx = \frac{1}{2}\int_{-\infty}^{\infty} x^2 e^{-|x|} dx = \frac{1}{2} \times 2\int_{0}^{\infty} x^2 e^{-x} dx = 2! = 2$$ 50.

Uncertainty in $\hat{x}$ is given by $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{2}$

Now, $\Delta x\, \Delta p|_{\min} = \hbar$ $\therefore$ $\Delta p_{\min} = \frac{h}{\Delta x} = \frac{h}{2\pi\sqrt{2}}$

**Correct option is (b)**

51.

48. $\frac{d^3y}{dx^3} + \frac{1}{x}\frac{d^2y}{dx^2} + \frac{1}{x^2}\frac{dy}{dx} = 0 \Rightarrow x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

Let $z = \ln x \Rightarrow \frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} = \frac{1}{x}\frac{dy}{dz}$

Also, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dz}\right) = -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d^2y}{dz^2} \times \frac{dz}{dx} = -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2y}{dz^2}$

Also, $\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)\right) = -\frac{2}{x^3}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right) + \frac{1}{x^2}\left(\frac{d^3y}{dz^3} - \frac{d^2y}{dz^2}\right) \times \frac{1}{x}$

$$= \frac{1}{x^3}\frac{d^3y}{dz^3} - \frac{3}{x^3}\frac{d^2y}{dz^2} + \frac{2}{x^3}\frac{dy}{dz}$$

Therefore, the given equation becomes

$$\left(\frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz}\right) + \left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right) + \frac{dy}{dz} = 0 \Rightarrow y''' - 2y'' + 2y' = 0$$

The auxilliary equation for this differential equation is

$$m^3 - 2m^2 + 2m = 0 \Rightarrow m(m^2 - 2m + 2) = 0 \Rightarrow m = 0,\ m = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$$

$$\therefore\ y(z) = C_1 + C_2 e^{(1+i)z} + C_3 e^{(1-i)z}$$

or $y(x) = C_1 + C_2 x^{1+i} + C_3 x^{1-i}$ $\quad \left(\because\ z = \ln x \Rightarrow x = e^z\right)$

**Correct option is (a)**

49. Given : $L = \left(\frac{dx}{dt}\right)^3 + x^3 = \dot{x}^3 + x^3$

The Euler equation of motion,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\Rightarrow \frac{d}{dt}(3\dot{x}^2) - 3x^2 = 0 \Rightarrow 2\dot{x}\ddot{x} - x^2 = 0 \Rightarrow \ddot{x} = \frac{x^2}{2\dot{x}} \Rightarrow \frac{d^2x}{dt^2} = \frac{1}{2}\frac{x^2}{dx/dt}$$

**Correct option is (c)**

50. The most general solution to the wave equation

$$\frac{\partial^2 F}{\partial x^2} = \frac{\partial^2 F}{\partial t^2}$$

This can be written as a linear combination of waves travelling towards left and right.

$$\therefore\ F(x,t) = h(x+t) + g(x-t)$$

**Correct option is (a)**

51. Let $F(x,t) = e^{\sqrt{k}x + 2kt} + e^{-\sqrt{k}x + 2kt}$. Then,

$$\frac{\partial F}{\partial x} = \sqrt{k}\left[e^{\sqrt{k}x+2kt} - e^{-\sqrt{k}x+2kt}\right] \text{ and } \frac{\partial^2 F}{\partial x^2} = k\left[e^{\sqrt{k}x+2kt} + e^{-\sqrt{k}x+2kt}\right]$$

So, $\frac{\partial^2 F}{\partial x^2} = kF(x,t)$

Similarly, we have

$$\frac{1}{2}\frac{\partial F}{\partial t} = \frac{1}{2}(2k)F(x,t) = kF(x,t)$$

**Correct option is (b)**

52. For $2 \times 2$ matrix $A$

$$\left.\begin{aligned} \text{Tr}(A) &= \lambda_1 + \lambda_2 = 0 \\ \det(A) &= \lambda_1 \lambda_2 = 0 \end{aligned}\right\} \text{given in question}$$

The only solution to these simultaneous equations is $\lambda_1 = \lambda_2 = 0$.

**Correct option is (b)**

53. $$A = \begin{pmatrix} 3 & 2i & 1 \\ -2i & 4 & 1+2i \\ 1 & 1-2i & 5 \end{pmatrix} \Rightarrow \hat{A}^{\dagger} = \begin{pmatrix} 3 & 2i & 1 \\ -2i & 4 & 1+2i \\ 1 & 1-2i & 5 \end{pmatrix}$$

This is a Hermitian matrix since $A^{\dagger} = A$. For a Hermitian matrix, all eigenvalues are real.

**Correct option is (c)**

54. We know that, Sum of the eigenvalues of a matrix = Trace of the matrix $\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 8$

and product of the eigenvalues of the matrix = Determinant of the matrix $\Rightarrow \lambda_1 \lambda_2 \lambda_3 \lambda_4 = -24$

Given: $\lambda_3 = 3$ and $\lambda_4 = 4$

$\therefore \quad \lambda_1 + \lambda_2 = 1, \ \lambda_1 \lambda_2 = -2$

$$\Rightarrow \lambda_1 - \frac{2}{\lambda_1} = 1 \Rightarrow \lambda_1^2 - \lambda_1 - 2 = 0 \Rightarrow (\lambda_1 - 2)(\lambda_1 + 1) = 0$$

$$\Rightarrow \left.\begin{aligned} \lambda_1 &= 2, -1 \\ \lambda_2 &= -1, 2 \end{aligned}\right\} \Rightarrow (\lambda_1, \lambda_2) = (2, -1)$$

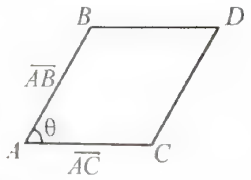
**Correct option is (b)**

55. $\vec{A} = y\hat{i} + z\hat{j} + x\hat{k}$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & z & x \end{vmatrix} = \hat{i}(-1) + \hat{j}(-1) + \hat{k}(-1) = -\hat{i} - \hat{j} - \hat{k}$$

**Correct option is (b)**

56. $\overrightarrow{AB} \times \overrightarrow{AC}$ = Area of parallelogram $ABDC$




**Correct option is (b)**

57. $f(x, y, z) = c$

The gradient to a surface at a point is along the outward normal to the surface at that point.

**Correct option is (c)**

58. $2y = x^2 \Rightarrow \dfrac{dy}{dx} = x$

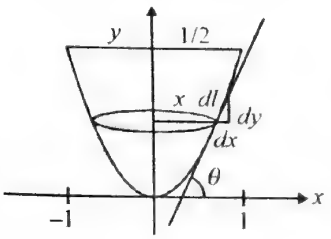
$dA = 2\pi x\, dl$

or $A = 2\pi \int x\, dl$

$= 2\pi \int x\left(\dfrac{dy}{\sin\theta}\right)$

$= 2\pi \times \int \dfrac{\sqrt{1+x^2}}{x} \times x\, dy \qquad \left[\sin\theta = \dfrac{dy}{\sqrt{dx^2+dy^2}} = \dfrac{x}{\sqrt{1+x^2}}\right]$

$= 2\pi \times \int_0^{1/2} \sqrt{(1+2y)}\, dy$




$\Rightarrow A = \dfrac{2\pi}{2} \times \dfrac{2}{3}(1+2y)^{3/2}\Big|_0^{1/2} = \dfrac{2\pi}{3}\left[2^{3/2} - 1\right] \Rightarrow A = \dfrac{2\pi}{3}\left(2^{3/2} - 1\right)$

**Correct option is (c)**

59. According to Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad \left(f(x) = \frac{1}{x} - a\right)$$

$$\Rightarrow x_{n+1} = x_n - \frac{\left(\dfrac{1}{x_n} - a\right)}{\left(-\dfrac{1}{x_n^2}\right)} \Rightarrow x_{n+1} = x_n + x_n - a x_n^2 \Rightarrow x_{n+1} = 2x_n - a x_n^2$$

**Correct option is (b)**

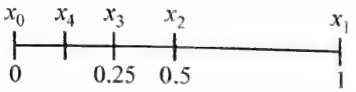
60. Iteration method is a method in which we use an initial Guess and iterate it a number of times to arrive at a converged solution.
Gauss elimination method is not an iterative method for finding solutions.
**Correct option is (c)**

61. Accuracy in iteration method is given by

$n = \log_2\left(\dfrac{\varepsilon_0}{\varepsilon}\right)$, where $n$ = number of iterations

$\varepsilon_0$ = initial bracket size

$\varepsilon$ = accuracy after after $n$ iterations




For $n = 10$

$$\Rightarrow 10 = \log_2\left(\frac{1}{\varepsilon}\right) \Rightarrow \varepsilon = 2^{-10} \Rightarrow \varepsilon = 0.0009$$

So, accuracy is upto 3 decimal places.
**Correct option is (b)**

62. We have electric field;

$$E = 100\cos(10^9 t - 4x)$$

Compare with the standard electric field equation.

$$E = A\cos(\omega t - kx)$$

we get, $\omega = 10^9 \sec^{-1}$; $k = 4m^{-1}$

$\therefore$ The speed of wave $v = \dfrac{\omega}{k} = \dfrac{10^9}{4} = 2.5\times10^8\ m/\sec$

**Correct option is (b)**

63. We know that the displacement vector is given by

$$\vec{D} = \varepsilon_0\vec{E} + \vec{p} = \varepsilon_0\vec{E} + \varepsilon_0\chi\vec{E}$$

$$\Rightarrow \quad \chi\vec{E} = \left(\frac{\vec{D}}{\varepsilon_0} - \vec{E}\right)$$

In CGS unit $\dfrac{1}{4\pi\varepsilon_0} = 1 \Rightarrow \varepsilon_0 = \left(\dfrac{1}{4\pi}\right)$

$$\therefore \quad \chi\vec{E} = \left(\vec{D} - \frac{\vec{E}}{4\pi}\right)4\pi$$

**Correct option is (*)**

64. The Poynting vector has dimension of (Energy/Area)

**Correct option is (a)**

65. Phase difference $\delta = \dfrac{2\pi}{\lambda}\Delta x = \dfrac{2\pi}{1.0}\times 0.5\times10^{-2}$ m (rad)

In degree $\delta = \dfrac{360^\circ}{2\pi}\times 2\pi\times0.5\times10^{-2} \Rightarrow \delta = 1.8^\circ$

**Correct option is (d)**

66. Oscillation of the pendulum with large amplitude is periodic, but not SHM.

**Correct option is (b)**

67. Let $x = A\sin\omega_0 t$

Time period $T_0 = \dfrac{2\pi}{\omega_0} = 4\sec$ (given)

$$\text{P.E.} = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2\cdot A^2\sin^2\omega_0 t \qquad \ldots(1)$$

$$\text{K.E.} = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega_0^2A^2\cos^2\omega_0 t$$

The difference in energy,

$\Delta E$ = K.E. – P.E.

$$= \frac{1}{2}m\omega_0^2A^2\left[\cos^2\omega_0 t - \sin^2\omega_0 t\right]$$

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$$= \frac{1}{2} m\omega_0^2 A^2 \cdot \cos 2\omega_0 t$$

$$= E_0 \cos 2\omega_0 t$$

i.e., difference in energy oscillates with frequency $2\omega_0$

Time period $T = \frac{2\pi}{2\omega_0} = \frac{T_0}{2} = 2\,\text{sec}$

**Correct option is (b)**

68. Fundamental frequency of one end close pipe is $f_c = \frac{v}{4L}$.

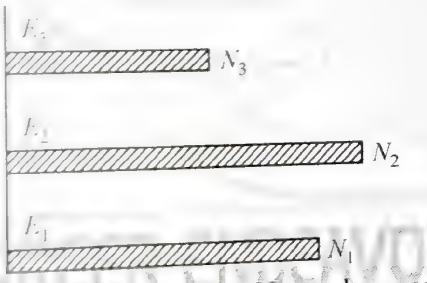
Let length of both end open pipe is $l$, then $f_0 = \frac{v}{2l}$. $\because \quad f_0 = f_c$

$$\frac{v}{4L} = \frac{v}{2l} \Rightarrow \frac{l}{L} = 2:1$$

**Correct option is (d)**

69. Splitting of spectral lines in the Stark effect depends on principal quantum number.
**Correct option is (d)**

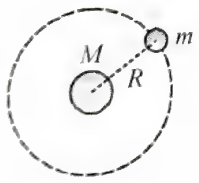
70. For laser emission the required the condition is population inversion. That says the number atoms in higher energy state will be larger than the number of atoms in ground state.




In this question it has been given that $N_2 > N_1 > N_3$, so laser emission will take place for transition from $E_2 \rightarrow E_1$.
**Correct option is (a)**

71. Question is not correct.

72. For circular motion, $\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v^2 = \frac{GM}{R}$




$$T = \frac{2\pi R}{v} = \frac{2\pi R}{(GM/R)^{1/2}} = \frac{2\pi}{\sqrt{GM}} R^{3/2}$$

$T \propto R^{3/2}$

**Correct option is (c)**

73. Given : $\sum_i^N \vec{F}_i = 0$

or, $\sum_i^N \frac{d\vec{p}_i}{dt} = 0$

or, $\frac{d}{dt}\sum_i^N (\vec{p}_i) = 0$

or, $\sum_i^N (\vec{p}_i) = \text{constant}$

Therefore, the linear momentum of the system is constant.

**Correct option is (c)**

74. Power, $P = \frac{|\Delta E|}{\Delta t} = \frac{\left|0 - \frac{1}{2} I\omega^2\right|}{t} = \frac{I\omega^2}{2t}$

**Correct option is (a)**

75. Rod's length as observed by observer at rest,

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

Putting the values of $L_0 = 1\,\text{m}$, $v = 0.8c$, we get

$L = 1 \times \sqrt{1 - 0.8^2} = 0.6\,\text{m}$

**Correct option is (b)**

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# PAPER : DU 2015
## PHYSICS-PH

**INSTRUCTIONS:**

(*i*) This test booklet contains 75 Objective type questions.

(*ii*) Each question carries **4 marks. 1 mark** will be deducted for each incorrect answer. Only one box should be marked and the marking of more than one box will be treated as wrong answer.

## OBJECTIVE QUESTIONS

1. A free electron and a free proton of the same energy have their de Broglie wavelengths in the ratio

(a) $\lambda_e / \lambda_p = (m_p / m_e)^{1/2}$ (b) $\lambda_e / \lambda_p = (m_e / m_p)^{1/2}$

(c) $\lambda_e / \lambda_p = (m_p / m_e)$ (d) $\lambda_e / \lambda_p = (m_e / m_p)$

2. The wave function of the particle is given by a linear combination of two of its energy states $\psi_1$ and $\psi_2$ (with energy eigen values $E_1$ and $E_2$ respectively) as $\psi = \psi_1 / a + \psi_2 / b$. If a measurement of energy is made, the probability of finding particle energy to be $E_1$ is

(a) $|b|^2 / [|a|^2 + |b|^2]$ (b) $|a|^2 / [|a|^2 + |b|^2]$

(c) $b/(a+b)$ (d) $a/(a+b)$

3. A particle of mass $m$ is in a one-dimensional potential given by $V(x) = \infty$ for $x < 0$ and $V(x) = kx^2/2$ for $x > 0$. The first excited state has an energy

(a) $(3/2)\hbar\sqrt{(k/m)}$ (b) $(7/2)\hbar\sqrt{(k/m)}$

(c) $(1/2)\hbar\sqrt{(k/m)}$ (d) $(5/2)\hbar\sqrt{(k/m)}$

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4. The wave function for a particle is given by $\psi = Ae^{-|x|}$, where A is a constant. This implies that the potential

(a) is differentiable everywhere

(b) is continuous everywhere but not differentiable at $x = 0$

(c) has discontinuity at $x = 0$

(d) has discontinuities at $x = \pm 1$ but differentiable every else

5. The electric field E and magnetic field B in frame 1 are parallel and related by $E = cB$. If the electric and magnetic fields in another frame are E′ and B′ respectively, the angle $x$ between E′ and B′ is given by $\cos(x)$ equals,

(a) B/B′ (b) B′/B (c) $(B/B')^{1/2}$ (d) $(B'/B)^{1/2}$

6. A linearly polarized electromagnetic wave is incident on a quarter wave plate. The emerge wave will in general be
(a) linearly polarized
(b) elliptically polarized
(c) unpolarized
(d) mixture of linearly polarized and unpolarized wave

7. The magnitude of the Poynting vector gives the
(a) energy transported per unit area per unit time
(b) energy density of the electromagnetic wave
(c) angular momentum density of the electromagnetic wave
(d) angular momentum transported per unit area per unit time

8. An unpolarized ray of light in a medium with refractive index $n_1$ is incident on a plane glass surface with refractive index $n_2 (n_2 > n_1)$. The intensity of the reflected light is minimum when the angle of incidence $i$ is given by
(a) $\tan(i) = n_1 / n_2$ (b) $\tan(i) = n_2 / n_1$
(c) $\tan(i) = 1 / n_1 n_2$ (d) $\tan(i) = n_1 n_2$

9. Assuming LS coupling, the lowest energy level for carbon (Z = 6) is
(a) $^1S_0$ (b) $^3P_0$ (c) $^1D_2$ (d) $^1S_1$

10. Rutherford bombarded 7.7 MeV $\alpha$ particles on $^{14}N_7$ to initiate the nuclear reaction $^{14}N(\alpha, p)^{17}O$. The height of Coulomb barrier faced by $\alpha$ particle is about (Given $r_o$ = 1.2 fm)
(a) 10.6 MeV (b) 2.1 MeV (c) 4.2 MeV (d) 8.4 MeV

11. Based on liquid drop model, the most stable nucleus for isobars of A = 75 has proton number (Z) equal to (Given: $a_c$ = 0.7 MeV and $a_s$ = 23.0 MeV)
(a) 28 (b) 33 (c) 35 (d) 37

12. The probability of electrons being captured by the nucleus is maximum for
(a) K shell electrons
(b) L shell electrons
(c) M shell electrons
(d) Electrons in outermost orbits, independent of which shell they come from

13. An electronic device exhibiting a negative resistance characteristic is the
(a) Zener diode (b) UJT (c) JFET (d) BJT

14. Role of the resistance ($R_E$) connected with the Emitter of an NPN transistor in the CE amplifier circuit is
(a) to fix the Q point at the center of the load line
(b) to increase the bandwidth of the amplifier
(c) to provide a path to the ac signal to ground
(d) to provide a negative feedback for dc bias stabilization

15. The $n$-channel JFET having pinch-off voltage of –5V shows a transconductance of 1mA/V when the applied gate to source voltage ($V_{GS}$) is –3V. The maximum value of transconductance (in mA/V) for JFET will be

(a) 1.5 (b) 2.0 (c) 2.5 (d) 3.0

16. The ideal OP-AMP with input resistance $R_i$, output resistance $R_o$ and gain A, has

(a) $R_i = \infty, A = \infty, R_o = 0$ (b) $R_i = 0, A = \infty, R_o = 0$

(c) $R_i = \infty, A = \infty, R_o = \infty$ (d) $R_i = 0, A = \infty, R_o = \infty$

17. The information in STACK in 8085 microprocessor follows

(a) FIFO (b) LIFO (c) FILO (d) random access

18. The T-states required for the opcode fetch in 8085 microprocessor and the execution time required for opcode fetch if the clock frequency is 2 MHz are respectively.

(a) $4T$, $2\mu s$ (b) $8T$, $4\mu s$ (c) $2T$, $2\mu s$ (d) $8T$, $2\mu s$

19. Assume the accumulator content in 8085 microprocessor is 00H and flag Z = 1. The content of accumulator (A) and Z flag after the execution of instruction MVI A, 05H are

(a) A = 05H, Z = 1 (b) A = 05H, Z = 0

(c) A = 00H, Z = 1 (d) A = 00H, Z = 0

20. Which of the following is not an infinite loop?

(a)
```
int i =1;
while (1)
{
i++;
}
```

(b)
```
for (;;);
```

(c)
```
int True = 0, False;
while (True)
{
False = 1;
}
```

(d)
```
int y, x = 0;
do
{
y = x;
} while (x == 0);
```

21. For the equation $x^3 - 2x - 5 = 0$ the solution by Newton Rapshon Method is about (Given $x_0$ = 2)

(a) 2.09983 (b) 2.07065

(c) 2.09455 (d) 2.14122

22. The real root of the equation $\cos x - 3x + 1 = 0$ corrected to four decimal places by the method of iteration is about

(a) 0.8071 (b) 0.6024

(c) 0.7214 (d) 0.6071

23. It is known that the root of the function $f(x)$ lies between 0 and 1. After 10 iterations of the bisection method, the approximate accuracy of the root will be

(a) correct up to 2 places of decimal (b) correct up to 3 places of decimal

(c) correct up to 4 places of decimal (d) correct up to 5 places of decimal

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24. The order of truncation error in second-order Runge-Kutta method for finding the solution of ordinary differential equations is
(a) 1 (b) 2 (c) 3 (d) 4

25. During inelastic collision of two bodies, which of the following is conserved?
(a) total linear momentum only (b) total kinetic energy only
(c) both linear momentum and kinetic energy (d) neither linear momentum nor kinetic energy

26. An earth satellite is kept moving in its orbit by the centripetal force provided by
(a) the gravitational attraction of the earth on the satellite
(b) the rocket engine attached to the satellite
(c) the gravitational attraction of the sun on the satellite
(d) the radio waves sent by the ground station to the satellite

27. A sphere rolls on a horizontal plane without slipping. The percentage of kinetic energy which is rotational is about
(a) 58% (b) 50% (c) 18% (d) 28%

28. A muon, whose life-time at rest is $2 \times 10^{-6}$ sec, is travelling through the laboratory at three-fifth of the speed of light. It will last in
(a) $2 \times 10^{-6}$ sec (b) $2.5 \times 10^{-6}$ sec (c) $3.5 \times 10^{-6}$ sec (d) $3.0 \times 10^{-5}$ sec

29. In an AC circuit, the voltage V and current I are given by:
$V = 100 \sin(100t)$ volts
$I = 100 \sin(100t + \pi/3)$mA
The power dissipated in the circuit is
(a) $10^4$ W (b) 10 W (c) 2.5 kW (d) 2.5 W

30. When a test charge is brought in from infinity along the perpendicular bisector of an electric dipole, the work done is
(a) positive (b) negative (c) zero (d) infinity

31. The ratio of magneto-motive force round the circuit to the magnetic flux is referred as
(a) reactance (b) conductance (c) resistivity (d) reluctance

32. A conducting circular loop is placed in a uniform magnetic field B = 0.04 T with its plane perpendicular to the field. Somehow the radius of the loop starts shrinking at a constant rate of 0.5 mm/sec. The induced emf in the loop at an instant, when the radius is 2cm, will be
(a) 5.0 μV (b) 2.5 μV (c) 10.0 μV (d) 25.0 μV

33. Consider a Boolean expression $F = (B + BC)(B + \bar{B}C)(B + D)$
The minimum number of NAND gates required to implement this 'F' will be
(a) 0 (b) 2 (c) 3 (d) 4

34. The number of comparators in a 4-bit flash A to D converter is
(a) 4 (b) 5 (c) 15 (d) 16

35. The functional difference between SR flip-flop and JK flip-flop is that
(a) JK flip-flop has a feedback path
(b) JK flip-flop can accept both inputs as 1
(c) JK flip-flop does not require an external clock pulse
(d) JK flip-flop is faster than SR flip-flop

36. The binary number corresponding to a decimal number 9.625 can be expressed as
(a) 1100.101 (b) 1001.110 (c) 1001.101 (d) 1100.011

37. The velocity of a traveling wave on a string under fixed tension
(a) does not change when frequency varies (b) does not change when wavelength varies
(c) decrease with increase of frequency (d) decrease with decrease in wavelength

38. Two springs of force constant $K_1$ and $K_2$ are arranged in a parallel arrangement and a mass '$m$' is suspended from it. The arrangement is equivalent to a single spring of constant K given by
(a) $K_1 - K_2$ (b) $K_1 + K_2$ (c) $K_1K_2/(K_1+K_2)$ (d) $K_1/K_2$

39. Two vibrating strings of lengths L and 3L and radii 3R and R, respectively, are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency $\nu_1$ and the other with frequency $\nu_2$. If the two strings are made of same material then the ratio $\nu_1/\nu_2$ is given by
(a) 1 (b) $3^{1/2}$ (c) 3 (d) 9

40. In a standing wave, the phase difference between any two points confined between adjacent node and antinode
(a) is always zero (b) is between $\pi/4$ and $\pi/2$
(c) is between $\pi/2$ and $\pi$ (d) changes periodically

41. If 'Z' is the atomic number and 'A' is the atomic weight of the element then according to Moseley's law, the frequency of the characteristic x-ray is proportional to
(a) $Z^2$ (b) $A^2$ (c) $Z^{3/2}$ (d) $A^{1/2}\,Z^{1/3}$

42. In M.K.S units, Bohr magneton is given by
(a) em/4h (b) eh/$2\pi$m (c) eh/$4\pi$m (d) mh/2e$\pi$

43. The typical wavelengths emitted by diatomic molecules in pure vibrational and pure rotational transitions are respectively in the region of
(a) infrared and visible (b) visible and infrared
(c) infrared and microwave (d) microwave and infrared

44. The value of Einstein's coefficient $A_{21}$ for a two level atomic system is $4 \times 10^2$ sec$^{-1}$. The spontaneous life time is
(a) $5 \times 10^{-2}$ sec (b) $2.5 \times 10^{-3}$ sec
(c) $5 \times 10^{-3}$ sec (d) $2.5 \times 10^{3}$ sec

45. Two lenses of power +5D and –2D are in contact and are coaxially situated. The power of the combination is
(a) –3D (b) +3D (c) –7D (d) +7D

46. When a liquid is introduced between the lens and the plate in the Newton Ring apparatus, the diameter of the 15$^{th}$ ring changes from 2cm to 1.8cm. The refractive index of the liquid is
(a) 1.05 (b) 1.11 (c) 1.23 (d) 1.33

47. When the movable mirror of the Michelson interferometer is moved through a distance of 0.030 mm under the illumination of a wavelength 6000Å, the number of displaced fringes is
(a) 0 (b) 10 (c) 100 (d) 600

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48. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.00 mm wide and the resulting diffraction pattern is observed on a screen 2m away. The distance between the first dark fringe on either side of the central maximum is

(a) 0.6 mm (b) 1.2 mm (c) 1.8 mm (d) 2.4 mm

49. The spectroscopic Lande g-factor for a state $^2P_{1/2}$ is

(a) 1 (b) 2/3 (c) 3/2 (d) 2

50. The Debye temperature of a metal is 450 K. Its Debye frequency in Hz is of the order of

(a) $10^{11}$ (b) $10^{13}$ (c) $10^{16}$ (d) $10^{20}$

51. In the Davission-Germer experiment, the hump is most prominent when the electron is accelerated by

(a) 24V (b) 34V (c) 54V (d) 64V

52. The susceptibility of a diamagnetic material is about

(a) $-10^{-5}$ (b) $10^{-7}$ (c) $10^{2}$ (d) $10^{3}$

53. If $\sigma$ and E are the electrical conductivity and the applied field respectively, the heat developed across a conductor per unit volume per second is

(a) $\sigma E^2$ (b) $\sigma^2 E$ (c) $\sigma / E$ (d) $E^2/\sigma$

54. Transition temperature $T_c$ and critical field $H_c$ for a superconductor are related as

(a) $H_c = H_0\left[1-\frac{T}{T_c}\right]$ (b) $H_c = H_0\left[\frac{T}{T_c}-1\right]$

(c) $H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$ (d) $H_c = H_0\left[\left(\frac{T}{T_c}\right)^2-1\right]$

55. The vapour pressure (p), in mm of Hg, of solid ammonia is given by the relation: $\ln p = 23.01 - (3754/T)$, where T is the absolute temperature. The vapour pressure, in mm of Hg, of liquid ammonia is given by the relation $\ln p = 19.49 - (3063/T)$. The temperature of the triple point is about

(a) 191.3 K (b) 196.3 K (c) 206.3 K (d) 296.3 K

56. The 10g of ice at 0°C is converted into water at the same temperature. Take latent heat of fusion as 80 cal/g. The change in entropy in cal/K is about

(a) zero (b) 0.34 (c) 2.93 (d) 3.92

57. A quantity of dry air at 27°C is compressed suddenly to one-third of its volume (Take $\gamma$ to be 1.5 for dry air), change in temperature of the gas is about

(a) 179K (b) 199K (c) 209K (d) 219K

58. Assume that each copper atom contributes one free electron to the electron gas. The density of copper is $8.94\times10^3$ kg/m$^3$ and its atomic mass is 63.5 amu. The Fermi energy (in joule) in copper is of the order of

(a) $10^{-18}$ (b) $10^{-13}$ (c) $10^{13}$ (d) $10^{18}$

59. The entropy of the system in statistical mechanics is defined as ($k$ is the Boltzmann constant and $\Omega$ is the number of microstates accessible to the system)

(a) $S = k \ln \Omega$ (b) $S = (k/2)\ln\Omega$ (c) $S = 2k \ln\Omega$ (d) $S = k^2 \ln\Omega$

60. If $E_1$ is the energy of the lowest state of a one-dimensional potential box of length '$a$' and $E_2$ is the energy of the lowest state when the length of the box is '$a/2$', then

(a) $E_2 = 2E_1$ (b) $E_2 = E_1/2$ (c) $E_2 = E_1/4$ (d) $E_2 = 4E_1$

61. The white dwarf stars are stable due to

(a) electron degeneracy pressure

(b) gravitational attraction

(c) heat generated by fusion against gravitational collapse

(d) fission

62. Let $\vec{r} = \hat{x}r\cos\omega t + \hat{y}r\sin\omega t$. For constant $r$ and $\omega$, calculate $\vec{r} \times \frac{d\vec{r}}{dt}$.

(a) $r^2\omega\hat{x}$ (b) $r^2\omega\hat{y}$ (c) $r^2\omega\hat{z}$ (d) $r^2\omega\cos(2\omega t)\hat{z}$

63. The integral $\int_{-\infty}^{+\infty} x^{2n}e^{-x^2}dx$ is equal to

(a) $\Gamma(n+1)$ (b) $\Gamma(n-1)$ (c) $\Gamma\left(n-\frac{1}{2}\right)$ (d) $\Gamma\left(n+\frac{1}{2}\right)$

64. Let $F(\omega)$ be the Fourier transform of $f(t)$ and $G(\omega)$ be the Fourier transform of $g(t) = f(t+a)$. The $G(\omega)$ and $F(\omega)$ are related with one another as

(a) $G(\omega) = e^{-i(a+t)\omega}F(\omega)$ (b) $G(\omega) = e^{-ia\omega}F(\omega)$

(c) $G(\omega) = e^{+i\omega t}F(\omega)$ (d) $G(\omega) = e^{+i\omega(t+a)}F(\omega)$

65. A finite amount of heat released at $t = 0$ spreads uniformly in all directions from the origin in a medium initially at zero temperature everywhere. The spatial and temporal temperature distribution is given as $\frac{C}{\sqrt{t^3}}e^{-r^2/(4a^2t)}$ where $a$ and $C$ are some constants. The time it takes for the temperature at distance '$r$' from the origin to attain its maximum value, is

(a) $\frac{r^2}{6a^2}$ (b) $\frac{r^2}{4a^2}$ (c) $\frac{3r^2}{8a^2}$ (d) $\frac{r^2}{a^2}$

66. The Euler-Lagrangian equation for a system is given as $\ddot{x} = -\gamma\dot{x} - \frac{k}{m}x$. Which of the following Lagrangians will describe this system?

(a) $L = \frac{1}{2}\exp(\gamma t)(m\dot{x}^2 + kx^2)$ (b) $L = \frac{1}{2}\exp(\gamma t)(m\dot{x}^2 - kx^2)$

(c) $L = \frac{1}{2}\exp(-\gamma t)(m\dot{x}^2 + kx^2)$ (d) $L = \frac{1}{2}\exp(-\gamma t)(m\dot{x}^2 - kx^2)$

67. The imaginary part of an analytic function in Cartesian coordinate is $2xy$. The real part is given by

(a) $x^2 + y^2 = a^2$ (b) $x^2 - y^2 = a^2$

(c) $-x^2 + y^2 = a^2$ (d) $-x^2 - y^2 = a^2$

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68. The Laurent series of a function $f(z)$ about $z=2$ is given by

$$f(z)=\frac{1}{2(z-2)^3}-\frac{1}{4(z-2)^2}+\frac{1}{8(z-2)}-\frac{1}{16}+\frac{z-2}{32}-\ldots..$$

The order of the pole and the value of the residue are

(a) Order = 3 and Residue = $\frac{1}{2}$

(b) Order = 2 and Residue = $-\frac{1}{4}$

(c) Order = 1 and Residue = $\frac{1}{8}$

(d) Order = 3 and Residue = $\frac{1}{8}$

69. The Bessel function $J_1(x)$ has its first zero at $x = 3.8317$. The intensity of light of wavelength $\lambda$ in the diffraction pattern from a circular aperture of size '$a$' at angle $\alpha$ is proportional to $\left\{J_1\left(\frac{2\pi a}{\lambda}\sin\alpha\right)\right\}^2$. The angle $\alpha$ at which the intensity will fall to zero for a typical wavelength of $5.5 \times 10^{-7}$ m and $a = 0.5$cm, is about

(a) 14 sec (b) 23 minutes (c) 24 degrees (d) 7 radians

70. Which of the following statements is true for a square matrix $A$?

(a) If $A^2 = 0$, it necessarily implies that $A = 0$

(b) If A is real and orthogonal, its eigenvalues will always be real

(c) If A is hermitian, its diagonal entries are always real

(d) If A is anti-hermitian (skew-hermitian), its diagonal entries are always zero

71. The magnitude of $\oint \vec{r}\times\overline{dr}$ over the perimeter of an ellipse described by $\vec{r}=\hat{x}a\cos\theta+\hat{y}b\sin\theta$, is

(a) $\pi ab/2$ (b) $\pi ab$ (c) $2\pi ab$ (d) $ab$

72. In which quadrants of the complex plane, is the function $f(z)=|x|-i|y|$ analytic?

(a) Only in the fourth quadrant (b) Only in the first and third quadrants

(c) Only in the second and fourth quadrants (d) In all the quadrants

73. The Laplace transforms $L\{f(t)\}=\int_0^\infty e^{-st}f(t)dt$ of $\cosh(kt)$ and $\sinh(kt)$ are given as

(a) $L\{\cosh kt\}=\frac{k}{s^2+k^2}$ and $L\{\sinh kt\}=\frac{s}{s^2+k^2}$

(b) $L\{\cosh kt\}=\frac{s}{s^2+k^2}$ and $L\{\sinh kt\}=\frac{k}{s^2+k^2}$

(c) $L\{\cosh kt\}=\frac{k}{s^2-k^2}$ and $L\{\sinh kt\}=\frac{s}{s^2-k^2}$

(d) $L\{\cosh kt\}=\frac{s}{s^2-k^2}$ and $L\{\sinh kt\}=\frac{k}{s^2-k^2}$

74. A vector $C_i$ and a tensor $C_{jk}$ are related as $C_i = \frac{1}{2} \in_{ijk} C_{jk}$ where $\in_{ijk}$ is the Levi-Civita symbol.

The tensor $C_{jk}$ is given by

(a) $\begin{pmatrix} 0 & C_3 & C_2 \\ C_3 & 0 & C_1 \\ C_2 & C_1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & C_3 & C_2 \\ -C_3 & 0 & -C_1 \\ -C_2 & C_1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & -C_3 & C_2 \\ C_3 & 0 & C_1 \\ -C_2 & -C_1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & C_3 & -C_2 \\ -C_3 & 0 & C_1 \\ C_2 & -C_1 & 0 \end{pmatrix}$

75. Under a rotation by an angle $\theta$ about the $x$-axis, the vector that remains invariant is

(a) $2\hat{x} - \hat{y} + \hat{z}$ (b) $3\hat{z}$ (c) $5\sqrt{2}\hat{x}$ (d) $\sqrt{3}\hat{y}$

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# SOLVED PAPER : DU 2015

PHYSICS-PH

## OBJECTIVE QUESTIONS

1. Since, the free electron and the free proton have same energy, their kinetic energies are equal.

$$(\text{K.E})_p = (\text{K.E})_e \Rightarrow \frac{1}{2} m_p v_p^2 = \frac{1}{2} m_e v_e^2 \Rightarrow \frac{v_p}{v_e} = \left(\frac{m_e}{m_p}\right)^{1/2}$$

Now, $\lambda = \dfrac{h}{p}$

$$\therefore \frac{\lambda_e}{\lambda_p} = \frac{m_p v_p}{m_e v_e} = \frac{m_p}{m_e}\sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{m_p}{m_e}}$$

**Correct option is (a)**

2. Since, $\psi_1$ is the energy eigenstate with energy $E_1$, the probability of the energy being $E_1$ upon measurement is,

$$p = \frac{|\langle \psi | \psi_1 \rangle|^2}{\langle \psi | \psi \rangle} = \frac{(1/|a|)^2}{\dfrac{1}{|a|^2} + \dfrac{1}{|b|^2}} = \frac{1}{|a|^2} \times \frac{|a|^2 |b|^2}{|a|^2 + |b|^2} = \frac{|b|^2}{|a|^2 + |b|^2}$$

**Correct option is (a)**

3. The given system is a half-harmonic oscillator whose energy states are given by,

$E_n = \left(n + \dfrac{1}{2}\right)\hbar\omega$ ; $n$ is odd

Therefore, for first excited state, $n = 3$, $\omega = \sqrt{k/m}$

Therefore, $E_3 = \dfrac{7}{2}\hbar\sqrt{k/m}$

**Correct option is (b)**

4. $\psi(x) = Ae^{-|x|}$

$$\therefore \lim_{x \to 0^+} \frac{d\psi}{dx} = \frac{d}{dx}(Ae^{-x})\bigg|_{x \to 0^+} = -A$$

$$\lim_{x \to 0^-} \frac{d\psi}{dx} = \frac{d}{dx}(Ae^{x})\bigg|_{x \to 0^-} = A$$

$$\therefore \lim_{x \to 0^+} \frac{d\psi}{dx} \neq \lim_{x \to 0^-} \frac{d\psi}{dx} \Rightarrow \frac{d\psi}{dx} \text{ has discontinuity at } x = 0.$$

$\psi$ is not differentiable at $x = 0$, but is continuous everywhere.

**Correct option is (b)**

5. According to relativistic theory $\vec{E}\cdot\vec{B}$ remain invariant

so $\vec{E}'\cdot\vec{B}' = \vec{E}\cdot\vec{B}$

$\Rightarrow \quad E'\cdot B'\cos x = E\cdot B\cos(0) = EB$

$$\Rightarrow \quad \cos x = \frac{EB}{E'B'}$$

Also $E^2 - c^2B^2 = E'^2 - c^2B'^2$

$\Rightarrow \quad E' = cB'$

$$\therefore \quad \cos x = \frac{cB^2}{cB'^2} = \left(\frac{B}{B'}\right)^2$$

**Correct option is (*)**

6. When a linearly polarized light incident on the quarter wave plate at an angle

$\phi = 0, 90° \rightarrow$ the emerge wave will be linear polarized.

$\phi = 45°, 135° \rightarrow$ the emerge wave will be circular polarized.

For any other angle except $\phi = 0, 90°, 45°, 135°$ the emerge wave will be elliptically polarized.

**Correct option is (b)**

7. The time average Poynting vector gives the energy transportance per unit area and per unit time.

**Correct option is (a)**

8. When an unpolarized incident at Brewster angle, then the reflected light becomes *S*-polarized light. The intensity of the reflected light will be minimum.

If $i_B$ is the Brewster angle, then we can write,

$$\tan i_B = \left(\frac{n_2}{n_1}\right) \Rightarrow \tan i = \left(\frac{n_2}{n_1}\right) \quad (\because i_B = i)$$

**Correct option is (b)**

9. $^{12}C_6 : 1s^2\, 2s^2\, 2p^2$

So, $s_1 = \frac{1}{2}, \quad s_2 = \frac{1}{2}, \quad l_1 = 1, \quad l_2 = 1$

So possible values of $S$ and $L$ are:

$S = |s_1 - s_2|$ to $|s_1 + s_2| = 0, 1$

$L = |l_1 - l_2|$ to $|l_1 + l_2| = 0, 1, 2$

Taking LS coupling:

$$\begin{cases} S = 0 \\ L = 0, 1, 2 \end{cases} \qquad J = 0, 1, 2 \qquad {}^1S_0 \;\; {}^1P_1 \;\; {}^1D_2$$

$$\begin{cases} S = 1 \\ L = 0 \end{cases} \qquad J = 1 \qquad {}^3S_1$$

$$\begin{cases} S = 1 \\ L = 1 \end{cases} \qquad J = 0, 1 \qquad {}^3P_0 \;\; {}^3P_1$$

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$\begin{cases} S=1 \\ L=2 \end{cases}$ $\quad J=1,2,3 \quad$ $^3D_1 \ ^3D_2 \ ^3D_3$

According to Hund's rule

1. The highest multiplicity lies lowest.
2. For less than half filled, the level with lowest $J$ value lies lowest.

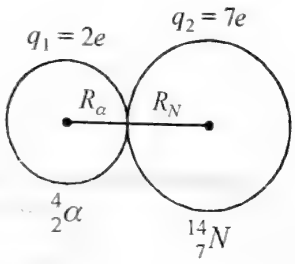
Therefore, $^3P_0$ is the lowest energy level for carbon.

**Correct option is (b)**

10. When a charge particle is bombarded on the nucleus. It has to ovecome Coulombic force before its nucleus touch each other.

Coulombic barrier is given by

$$E_c = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$




Here, $q_1 = 2e$, $q_2 = 7e$, $r = R_\alpha + R_N$

Radius of nucleus is $R = R_0 A^{1/3}$

$$\Rightarrow E_c = \frac{1}{4\pi\varepsilon_0} \times \frac{(2e)(7e)}{R_0\left[4^{1/3}+7^{1/3}\right]}$$

Putting $R_0 = 1.2$ fm, $\frac{e^2}{4\pi\varepsilon_0} = 1.44$ MeV fm, we get

$$E_c = 4.20 \text{ MeV}$$

**Correct option is (c)**

11. Binding energy based upon liquid drop model

$$\text{B.E} = a_v A - a_s A^{2/3} - a_c z(z-1)A^{-1/3} - a_{sys}\frac{(A-2z)^2}{A} + \delta$$

Binding energy is maximum for stable isobars $\frac{d_{BE}}{dz} = 0$

Solving we get,

$$\Rightarrow z_{min} = \frac{A}{2}\left[\frac{1}{1+\frac{1}{4}\frac{a_c}{a_{sys}}A^{2/3}}\right]$$

For $A = 75$, we get

$$z_{min} = \frac{75}{2}\left[\frac{1}{1+\frac{1}{4}\left(\frac{0.7}{23}\right)(75)^{2/3}}\right] = 33.03 \approx 33$$

**Correct option is (b)**

12. Electron capture is another form of beta decays. It is some times also called K-capture, because most of the electrons are captured from the K-shell, which is closest to the nucleus.
**Correct option is (a)**

13. In Unijunction Transistor (UJT) emitter current increases as emitter voltage decreases. So the slope of V vs. I curve is negative for that UJT exhibits a negative resistance characteristics.
**Correct option is (b)**

14. $A_V = \dfrac{A_I \cdot R_L}{R_i}$

$R_i = h_{ie} \underset{\text{with } R_E}{+} (1+h_{fe}) R_E$

$R_i = h_{ie} + (1+h_{fe}) R_E \leftarrow$ with $R_E$

$R_i = h_{ie} \leftarrow$ without $R_E$

Presence of $R_E$ increases $R_i$ (input impedance) and it decreases voltage gain.

$$\text{Gain} \propto \frac{1}{\text{Band width}}$$

$\Rightarrow A_V$ decreases and hence 'BW' increases.
**Correct option is (b)**

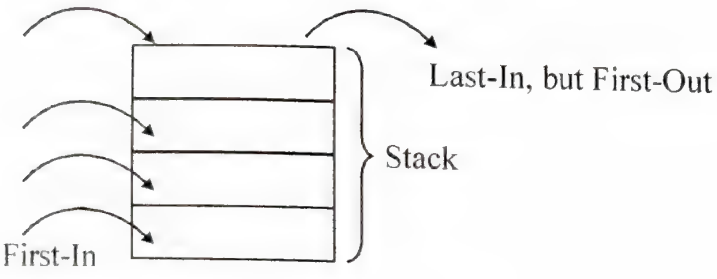
15. $g_m = g_{m_0}\left[1 - \dfrac{V_{GS}}{V_P}\right]; \quad g_{m_0} = \left|\dfrac{2I_{DSS}}{V_P}\right|$

$1 = g_{m_0}\left[1 - \dfrac{-3}{-5}\right]; \quad g_{m_0} = 2.5\,\dfrac{mA}{V}$

$g_{m_0}$ = maximum transconductance = 2.5 mA/V
**Correct option is (c)**

16. Input resistance $\leftarrow R_i \to \infty$
Output resistance $\leftarrow R_0 \to 0$
Open-loop-gain $\leftarrow A_{OL} \to \infty$
For ideal op-amp.
**Correct option is (a)**

17.




Last input first-out : LIFO
**Correct option is (c)**

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18. 99 % of opcode fetch for any instruction in microprocessor 8085 has '4T' opcode fetch.
'6T' for **"Return with condition"**
RZ Return zero
RNZ Return not zero
RC Return carry
RNC Return no carry
RPO Return parity odd
RPE Return parity even
RP Return positive
RM Return minus
'6T' for PUSH
SPHL
INX

Execution time = $4T = \frac{4}{f} \leftarrow 2\times10^6$

$t = 2\,\mu sec$

**Correct option is (a)**

19. Accumulator = $00_H$
Zero flag (ZF) = 1
MVI $A_0$ 05H
$A = 05_H$
Z·F = 1 → Remain same
**Correct option is (a)**
**Note:** Data transfer operation never effect condition of flag.

20. (a) while (1) condition always true → infinite loop.
(b) For (; ;); → no condition. Hence, infinite loop.
(c) True = 0; while true.
Since, true variable. Holds 0 which implies condition always false → not infinite loop.
**Correct option is (c)**

21. Given equation: $x^3 - 2x - 5 = 0 \Rightarrow f(x) = x^3 - 2x - 5 \Rightarrow f'(x) = 3x^2 - 2$
According to Newton-Raphson method,
**1st iteration**

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} \Rightarrow x_1 = 2 - \frac{(-1)}{10} = 2 + 0.1 = 2.1$$

**2nd iteration,**

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.1 - \frac{0.061}{11.23} = 2.094$$

**3rd iteration,**

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.094 - \frac{(-0.003)}{11.154} = 2.094$$

The solution converges to $x = 2.094$.
**Correct option is (c)**

22. $\cos x - 3x + 1 = 0 \Rightarrow \cos x = 3x - 1$

$\Rightarrow -1 \le 3x - 1 \le 1 \Rightarrow \boxed{0 \le x \le \frac{2}{3}}$

$\Rightarrow \boxed{0 \le x \le 0.6666}$

Now, $f(x) = \cos x - 3x + 1$

$\Rightarrow f(0.600) = \cos(0.6) - 3 \times 0.6 + 1 \approx 0.025 > 0$

$f(0.603) = \cos(0.603) - 3 \times 0.6 + 1 \approx 0.014 > 0$

$f(0.6075) = \cos(0.6075) - 3 \times 0.6075 + 1 \approx -0.0014 < 0$

$\Rightarrow$ The root of $f(x)$ should lie between 0.6030 and 0.6075

$\Rightarrow$ The root of $f(x)$ is $x = 0.6071$

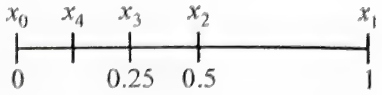
**Correct option is (d)**

23. Accuracy in iteration method is given by

$n = \log_2\left(\frac{\varepsilon_0}{\varepsilon}\right)$, where $n$ = number of iterations

$\varepsilon_0$ = initial bracket size

$\varepsilon$ = accuracy after after $n$ iterations




For $n = 10$, $\Rightarrow 10 = \log_2\left(\frac{1}{\varepsilon}\right) \Rightarrow \varepsilon = 2^{-10} \Rightarrow \varepsilon = 0.0009$

So, accuracy is upto 3 decimal places.

**Correct option is (b)**

24. Truncation error in second-order Runge-Kutta method is of second order.

**Correct option is (b)**

25. During inelastic collision of two bodies, total linear momentum is conserved and total kinetic energy is not conserved.

**Correct option is (a)**

26. An Earth satellite is kept moving in its orbit by centripetal force provided by the gravitational attraction of the Earth on the satellite.

**Correct option is (a)**

27. For pure rolling $v = \omega R$

Rotational kinetic energy,

$$\text{K.E}_{\text{rot}} = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}MR^2\omega^2$$

Total kinetic energy, $\text{K.E}_{\text{Total}} = \text{K.E}_{\text{trans.}} + \text{K.E}_{\text{rot}}$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}MR^2\omega^2$$

$$= MR^2\omega^2\left(\frac{1}{2} + \frac{1}{5}\right) = \frac{7}{10}MR^2\omega^2$$

Percentage of kinetic energy which is rotational,

$$= \frac{\text{K.E}_{\text{rot.}}}{\text{K.E}_{\text{total}}} \times 100 = \frac{\frac{1}{5}MR^2\omega^2}{\frac{7}{10}MR^2\omega^2} \times 100 = \frac{2}{7} \times 100 = 28.57\%$$

**Correct option is (d)**

28. Life time of a muon as observed from laboratory,

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Putting values of $\tau_0 = 2 \times 10^{-6}$ s, $v = \frac{3}{5}c$, we get

$$\tau = \frac{2 \times 10^{-6}}{\sqrt{1-\frac{9}{25}}} = 2 \times 10^{-6} \times \frac{5}{4} = 2.5 \times 10^{-6} \text{ s}$$

**Correct option is (b)**

29. $V = 100 \sin(100t)$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$

$$I = 100 \sin\left(100t + \frac{\pi}{3}\right)$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$

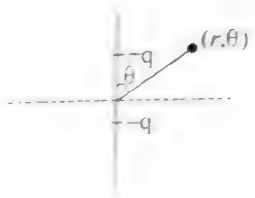
$\cos\phi$ = phase OR power factor

$$P = V_{rms}\, I_{rms} \cos\frac{\pi}{3} \text{ (put value)}$$

$$P = \frac{100}{\sqrt{2}} \cdot \frac{100}{\sqrt{2}} \cdot \frac{1}{2} = 2.5 \text{ kW}$$

**Correct option is (c)**

30. The potential at any point (r, θ), due to an electric dipole is given by




$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\left(\frac{P\cos\theta}{r^2}\right)$$

On the perpendicular bisector of an electric dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{P\cos 90^\circ}{r^2}\right) = 0$$

So, workdone to bring the test charge from infinity to any finite point along the perpendicular bisector is given by

$$W = (V_f - V_i)q = 0$$

**Correct option is (c)**

CAREER ENDEAVOUR

31. The ratio of magnetomotive force to the magnetic flux has dimension of reluctance.
**Correct option is (c)**

32. At certain time $t$, the area of the loop is $A = \pi r^2$

$\therefore$ The magnetic flux passing through the loop is given by $\phi = BA = B\pi r^2$

$\therefore$ The induced emf in the loop is given by

$$\text{emf} = \frac{-d\phi}{dt} = -2B\pi r\frac{dr}{dt} = 2 \times B\pi r\left(\frac{-dr}{dt}\right)$$

$$= 2\pi \times 0.04 \times 2 \times 10^{-2} \times 0.5 \times 10^{-3} = 2.5\,\mu V$$

**Correct option is (c)**

33. $F = (B + BC)(B + \bar{B}C)(B + D)$

$F = B(1 + C)(B + \bar{B})(B + C)(B + D)$

$F = B(B + C)(B + D)$

$F = (B + BC)(B + D)$

$F = B(1 + C)(B + D)$

$F = B(B + D)$

$F = B + BD = B(1 + D)$

$\boxed{F = B}$ simplified

| |
|---|
| $1 + x = 1$ |
| $a + bc = (a + b)(a + c)$ |
| Distributive |
| B ●——— ● F |

**Correct option is (a)**

34. Number of comparator for 'n' bit flash ADC is
Number of comparator $= 2^n - 1$, put $n = 4$
**Correct option is (c)**

35. Truth table of SR flip-flop

| S | R | Q | $\bar{Q}$ | |
|---|---|---|---|---|
| 0 | 0 | Q | $\bar{Q}$ | Hold |
| 0 | 1 | 0 | 1 | Reset |
| 1 | 0 | 1 | 0 | Set |
| 1 | 1 | 0 | × | Prohibited |

Truth table of JK flip-flop

| J | K | Q | $\bar{Q}$ | |
|---|---|---|---|---|
| 0 | 0 | Q | $\bar{Q}$ | Hold |
| 0 | 1 | 0 | 1 | Reset |
| 1 | 0 | 1 | 0 | Set |
| 1 | 1 | $\bar{Q}$ | Q | Complement |

**Correct option is (b)**

36.

| | | |
|---|---|---|
| 2 | 9 | 1 ↑ |
| 2 | 4 | 0 |
| 2 | 2 | 0 |
| | 1 | |

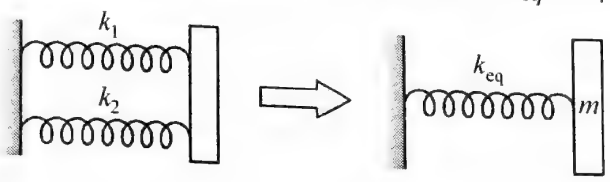
$9 = (1001)_2$

| | |
|---|---|
| $0.625 \times 2 = \mathbf{1}.25$ | 1 ↓ |
| $0.25 \times 2 = \mathbf{0}.50$ | 0 |
| $0.5 \times 2 = \mathbf{1}.00$ | 1 |
| $0.00 \times 2 = \mathbf{0}.00$ | 0 |

Therefore, the binary number corresponding to decimal number 9.625 is 1001.1010.
**Correct option is (c)**

37. Velocity of the travelling wave in a string is given by
$v = \sqrt{T/\mu}$, where $T$ = tension, $\mu$ = mass per unit length.
If tension is fixed, $v$ is fixed and is independent of frequency and wavelength.
**Correct options are (a) and (b)**

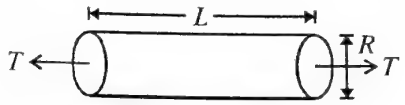
38. Equivalent spring constant of spring connected in parallel is $k_{eq} = k_1 + k_2$.




**Correct option is (b)**

39. Velocity of wave in string $v = \sqrt{T/\mu}$.

If $\rho$ be the density of the string material.




Mass of string $M = \rho V = \rho \pi R^2 L$

Mass per unit length $\mu = \dfrac{M}{L} = \rho \pi R^2$

Fundamental frequency $\nu = \dfrac{nv}{2L} = \dfrac{v}{2L}$ $(n = 1)$

$$\Rightarrow \nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{T}{\rho\pi R^2}} = \frac{1}{2LR}\sqrt{\frac{T}{\rho\pi}}$$

$$\Rightarrow \nu = \frac{k}{LR};$$

where $k = \dfrac{1}{2}\sqrt{\dfrac{T}{\rho\pi}}$

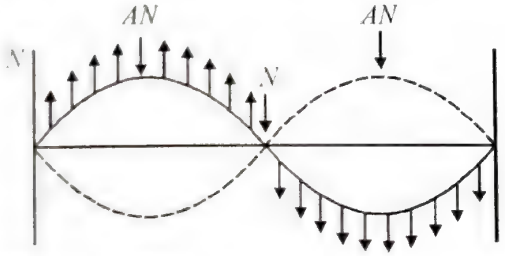
Let $L_1 = L$, $R_1 = 3R$ and $L_2 = 3L$, $R_2 = R$

$$\Rightarrow \frac{\nu_1}{\nu_2} = \frac{L_2 R_2}{L_1 R_1} = \frac{3L \cdot R}{L \cdot 3R} = 1$$

**Correct option is (c)**

CAREER ENDEAVOUR

40. Region of particle vibrating in adjacent antinode and node is shown in figure.




All the particle in a loop (between node to node) vibrate in same phase, i.e., all particle go up or down together as shown in figure.. Therefore, phase difference is always zero between antinode ($AN$) and node ($N$).

**Correct option is (a)**

41. According to Moseley's law, the frequency of the characteristics X-rays is given by

$\nu = B(z - \sigma)^2 \Rightarrow \nu \propto z^2$

**Correct option is (a)**

42. In M.K.S units, Bohr magneton is given by $\mu_B = \frac{eh}{4\pi m}$, where

$e$ = electron charge
$h$ = Planck's constant
$m$ = mass of an electron.

**Correct option is (c)**

43. The typical energy emitted by diatomic molecules in pure vibrational and pure rotational transistors are respectively of the order of $10^{-20}$ Joules and $10^{-23}$ Joules. These are the energy value corresponding infrared and microwave region of EM wave spectrum.

**Correct option is (c)**

44. Given the value of Einstein's co-efficient $A_{21} = 4 \times 10^2 \text{ sec}^{-1}$

Therefore, the spontaneous life time $\tau = \frac{1}{A_{21}} = \frac{10^{-2}}{4} \text{sec} = 2.5 \times 10^{-3} \text{ sec}$

**Correct option is (b)**

45. Equivalent power of lens in contact are given by

$$D = D_1 + D_2 \ldots.$$
$$= +5D - 2D$$
$$= +3D$$

**Correct option is (d)**

46. Diameter of Newton's ring is related with the wavelength

$D^2 = \frac{4m\lambda_0 R}{n}$; where $m$ is order of ring.

$\Rightarrow D^2 \propto \frac{\lambda_0}{n}$; $\lambda_0$ = wavelength in air

$n$ = refractive index of medium

$$\Rightarrow \left(\frac{D_{air}}{D_m}\right)^2 = \frac{n}{n_{air}} \Rightarrow \left(\frac{2}{1.8}\right)^2 = n \Rightarrow 1.23 = n$$

**Correct option is (c)**

47. For Michelson's interferometer $2d = N\lambda$,

where $d$ = change in distance between mirror

$N$ = number of fringe appear or disappear

$\lambda$ = wavelength of light

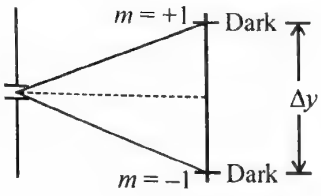
$$\Rightarrow N = \frac{2d}{\lambda} = \frac{2 \times 0.030 \times 10^{-3} \text{ m}}{6000 \times 10^{-10} \text{ m}} = 100$$

**Correct option is (c)**

48. In case of single slit, position of dark $y = \frac{m\lambda D}{a}$.

For first dark $m = \pm 1$, difference between position is

$$\Delta y = +\frac{\lambda D}{a} - \left(-\frac{\lambda D}{a}\right) = \frac{2\lambda D}{a} = \frac{2 \times 6000 \times 10^{-9}\ \text{m} \times 2\ \text{m}}{1.00 \times 10^{-3}\ \text{m}} = 2.4\ \text{mm}$$




**Correct option is (d)**

49. Given : Spectroscopic term $^2P_{1/2}$

Therefore, $\ell = 1,\ s = \frac{1}{2},\ j = \frac{1}{2}$

Therefore, the Lande g-factor is given by

$$g = 1 + \frac{j(j+1) - \ell(\ell+1) + s(s+1)}{2j(j+1)} = 1 + \frac{\frac{1}{2}\left(\frac{3}{2}\right) - 1(2) + \frac{1}{2}\left(\frac{3}{2}\right)}{2 \times \frac{1}{2} \times \frac{3}{2}}$$

$$= 1 + \frac{\frac{3}{4} + \frac{3}{4} - 2}{\frac{3}{2}} = 1 + \frac{\frac{3}{2} - 2}{\frac{3}{2}} = 1 - \frac{1}{3} = \frac{2}{3}$$

**Correct option is (b)**

50. We know the relation between Debye frequency and temperature is $h\nu_D = k_B \theta_D$

$$\Rightarrow \nu_D = \left(\frac{k_B \theta_D}{h}\right) = \frac{1.38 \times 10^{-23} \times 450}{6.626 \times 10^{-34}}\ \text{Hz}$$

$$= 93.8 \times 10^{11}\ \text{Hz} \approx 10^{13}\ \text{Hz}$$

**Correct option is (b)**

51. In the Davisson-Germer experiment, the hump is most prominent when the electron is accelerated by 54 V.

**Correct option is (c)**

52. The susceptibility of a diamagnetic material is $\chi < 0$.

**Correct option is (a)**

53. According Ohm's law, $V = IR \Rightarrow (J = \sigma E)$

And heat produce in the conductor per unit volume,

$$H = \frac{I^2 R}{V'} = \frac{I^2 \frac{1}{\sigma}\frac{L}{A}}{V'} = \frac{J^2 A^2 L}{\sigma A V'} = \frac{J^2}{\sigma} = \sigma E^2, \text{ (where volume of the metal } V' = AL)$$

**Correct option is (b)**

54. The relation between transition temperature $T_c$ and critical field of an superconductor can be written as

$$H(T) = H_0\left(1 - \frac{T^2}{T_c^2}\right)$$

**Correct option is (c)**

55. At triple point, we have

$$P_{\text{vapour}} = P_{\text{liquid}}$$

$$\Rightarrow 23.01 - \frac{3754}{T} = 19.49 - \frac{3063}{T}$$

$$\Rightarrow T = 196.3\ K$$

**Correct option is (b)**

56. The change in entropy is

$$\Delta S = \frac{mL}{T} = \frac{10 \times 80}{273} \simeq 2.93 \text{ cal/K}$$

**Correct option is (c)**

57. For sudden compression, the gas must undergo adiabatic process. So, we have

$$TV^{\gamma-1} = \text{constant}$$

$$\Rightarrow T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 300\left(\frac{V_0}{V_0/3}\right)^{1.5-1}$$

$$\Rightarrow T_f = 300\sqrt{3} = 519.6\ \text{K}$$

$$\therefore \quad \Delta T = 519.6 - 300.0 = 219.6\ \text{K}$$

**Correct option is (d)**

58. The fermi energy of metal is given by

$$E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3},$$

where $h$ = Planck constant,
$m$ = Mass of electron,
$n$ = Free electron density.
Given density of Cu

$$\rho = 8.94 \times 10^3\ \frac{\text{kg}}{\text{m}^3}$$

Atomic mass = 63.5 amu.
The number of free electrons per unit volume,

$$n = \left(\frac{\rho \times N_A}{M}\right) = \frac{8.94 \times 10^3 \times 6.062 \times 10^{26}}{63.5}/\text{m}^3$$

$$= 0.853 \times 10^{29}/\text{m}^3$$

$$\therefore \quad E_F = \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}}\left(\frac{3 \times 0.853 \times 10^{29}}{8\pi}\right)^{2/3}$$

$$= 7.069\ \text{eV} \simeq 10^{-18}\ \text{J}$$

**Correct option is (a)**

59. The statistical entropy is given by $S = k \ln \Omega$

**Correct option is (a)**

60. Energy of the $n^{th}$ state for a particle of mass '$m$' in a one-dimensional box of length $a$ is,

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2} \quad (n = 1, 2, 3, ...)$$

Here, $E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$, and $E_2 = \frac{\pi^2 \hbar^2}{2m(a/2)^2} = \frac{2\pi^2 \hbar^2}{ma^2} = 4E_1$

**Correct option is (d)**

61. White dwarf stars are stable due to electron degenaracy pressure.

**Correct option is (a)**

62. $\vec{r} = \hat{x} r\cos\omega t + \hat{y} r\sin\omega t \Rightarrow \frac{d\vec{r}}{dt} = -r\omega\left[\sin\omega t\,\hat{x} - \cos\omega t\,\hat{y}\right]$

$$\vec{r} \times \frac{d\vec{r}}{dt} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r\cos\omega t & r\sin\omega t & 0 \\ -r\omega\sin\omega t & +r\omega\cos\omega t & 0 \end{vmatrix} = \hat{z}\left(r^2\omega\cos^2\omega t + r^2\omega\sin\omega t\right) = r^2\omega\,\hat{z}$$

$$\Rightarrow \left|\vec{r} \times \frac{d\vec{r}}{dt}\right| = r^2\omega\,\hat{z}$$

**Correct option is (c)**

63. $\int_{-\infty}^{+\infty} x^{2n} e^{-x^2} dx = \Gamma\left(n + \frac{1}{2}\right)$

**Correct option is (d)**

64. $F(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} f(t)\,dt \qquad G(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} f(t+a)\,dt$

Taking $u = t + a \Rightarrow du = dt$

$$= \int_{-\infty}^{\infty} e^{i\omega(u-a)} f(u)\,du = \left[\int_{-\infty}^{\infty} e^{i\omega u} f(u)\,du\right] \times e^{-i\omega a}$$

$$\Rightarrow G(\omega) = e^{-i\omega a} F(\omega)$$

**Correct option is (b)**

65. $T(r,t) = \frac{c}{\sqrt{t^3}} e^{-r^2/(4a^2 t)}$

$$\Rightarrow \frac{\partial T}{\partial t} = c\left[-\frac{3}{2} r^{-5/2} + \left(-\frac{r^2}{4a^2}\right) \times \left(-\frac{1}{t^2}\right) t^{-3/2}\right] e^{-r^2/4a^2 t}$$

For maximum temperature,

$$\frac{\partial T}{\partial t} = 0 \Rightarrow t^{-7/2}\left(\frac{r^2}{4a^2}\right) - \frac{3}{2} t^{-5/2} = 0 \Rightarrow t^{-7/2}\left[\frac{r^2}{4a^2} - \frac{3}{2} t\right] = 0 \Rightarrow t = \frac{r^2}{6a^2}$$

**Correct option is (a)**

66. Equation of motion corresponding to Lagrangian given in option (b);

$$L = \frac{1}{2}e^{\gamma t}(m\dot{x}^2 - kx^2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}e^{\gamma t}(2m\dot{x})\right) - \frac{1}{2}e^{\gamma t}(-2kx) = 0 \Rightarrow \gamma e^{\gamma t} m\dot{x} + e^{\gamma t} m\ddot{x} + e^{\gamma t} kx = 0$$

$$\Rightarrow \ddot{x} = -\gamma\dot{x} - \frac{k}{m}x$$

**Correct option is (b)**

67. $v(x, y) = 2xy$

Here, $\frac{\partial v}{\partial y} = 2x, \quad \frac{\partial v}{\partial x} = 2y$

According to Cauchy-Riemann conditions for complex analytic function,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \Rightarrow \frac{\partial u}{\partial x} = 2x \Rightarrow u(x, y) = x^2 + f(y) \Rightarrow \frac{\partial u}{\partial y} = f'(y)$$

But $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \Rightarrow f'(y) = -2y \Rightarrow f(y) = -y^2 + c$

$\therefore \quad u(x, y) = x^2 - y^2 + c$

**Correct option is (b)**

68. $f(z) = \frac{1}{2(z-2)^3} - \frac{1}{4(z-2)^2} + \frac{1}{8(z-2)} - \frac{1}{16} + \frac{z-2}{32} - \ldots$

The highest negative power of $(z - 2)$ present in the Laurent series expansion is -3. Therefore, $z = 2$ is a pole of order 3 of $f(z)$.

Residue of $f(z)$ at $(z = 2)$ = Co-efficient of $\frac{1}{(z-2)}$ in the Laurent expansion of $f(z) = 1/8$

**Correct option is (d)**

69. $I = \left\{J_1\left(\frac{2\pi a}{\lambda}\sin\alpha\right)\right\}^2$

For the intensity to go to zero.

$$J_1\left(\frac{2\pi a}{\lambda}\sin\alpha\right) = 0 \Rightarrow \frac{2\pi a}{\lambda}\sin\alpha = 3.8317$$

$$\Rightarrow \sin\alpha = \frac{3.8317 \times 5.5 \times 10^{-7}}{2 \times 3.14 \times 0.5 \times 10^{-2}} \Rightarrow \alpha = 14\,\text{sec}$$

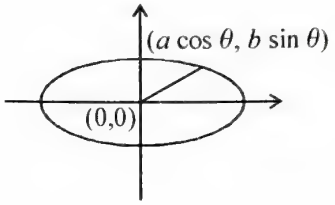
**Correct option is (a)**

70. (a) $A^2 = 0$ does not necessarily mean $A = 0$.
(b) If $A$ is real and orthogonal, its eigenvalues need not always be real.
(c) If $A$ is hermitian, $A^\dagger = A \Rightarrow a_{ii}^* = a_{ii} \Rightarrow$ diagonal entries are real.
(d) If $A$ is anti-hermitian, $A^\dagger = -A \Rightarrow a_{ii}^* = -a_{ii} \Rightarrow$ diagonal entries are purely imaginary or zero.
**Correct option is (c)**

71. $\vec{r} = a\cos\theta\,\hat{x} + b\sin\theta\,\hat{y}$

$d\vec{r} = (-a\sin\theta\,\hat{x} + b\cos\theta\,\hat{y})\,d\theta$

$$\vec{r}\times d\vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a\cos\theta & b\sin\theta & 0 \\ -a\sin\theta\,d\theta & b\cos\theta\,d\theta & 0 \end{vmatrix} = \hat{z}\,(a\,b)\,d\theta$$




$$\Rightarrow \oint \vec{r}\times d\vec{r} = (a\,b)\,\hat{z}\int_0^{2\pi} d\theta = 2\pi ab\,\hat{z} \Rightarrow \left|\oint \vec{r}\times d\vec{r}\right| = 2\pi ab$$

**Correct option is (c)**

72. $f(z) = |x| - i|y|$

In first quadrant, $f(z) = x - iy = \bar{z}$ (Non-analytic)
In second quadrant, $f(z) = -x - iy = -(x+iy) = -z$ (Analytic)
In third quadrant, $f(z) = -x + iy = -\bar{z}$ (Non-analytic)
In fourth quadrant, $f(z) = x + iy = z$ (Analytic)

So, $f(z)$ is analytic in second and fourth quadrants.
**Correct option is (c)**

73. $L(\cosh(kt)) = \dfrac{s}{s^2 - k^2} \Rightarrow L(\sinh(kt)) = \dfrac{k}{s^2 - k^2}$

These are standard Laplace transforms.
**Correct option is (d)**

74. $\varepsilon_{123}$ and its cyclic permutation have value = 1.
$\varepsilon_{213}$ and its cyclic permutations have value = –1.
$\varepsilon_{iij} = \varepsilon_{iii} = 0\ \forall\ i, j = 1, 2, 3$

$$C_i = \frac{1}{2}\varepsilon_{ijk}C_{jk} = -\frac{1}{2}\varepsilon_{ikj}C_{jk} = -\frac{1}{2}\varepsilon_{ijk}C_{kj} \Rightarrow C_{jk} = -C_{kj}$$

$C_i = \dfrac{1}{2}\varepsilon_{ijk}C_{jk}$ [In tensor notation, there will be a summation over the repeated indices $j$ and $k$]

For $i = 1$, $C_1 = \dfrac{1}{2}\varepsilon_{1jk}C_{jk} \Rightarrow C_1 = \dfrac{1}{2}\varepsilon_{123}C_{23} + \dfrac{1}{2}\varepsilon_{132}C_{32} \Rightarrow C_1 = \dfrac{1}{2}C_{23} - \dfrac{1}{2}C_{32}$

Therefore, $C_{23} = C_1$ and $C_{32} = -C_1$
Simlarly, we can find $C_{12} = C_3$, $C_{21} = -C_3$, $C_{13} = -C_2$, $C_{31} = C_2$

$$\therefore\ C_{jk} = \begin{pmatrix} 0 & +C_3 & -C_2 \\ -C_3 & 0 & C_1 \\ +C_2 & -C_1 & 0 \end{pmatrix}$$

**Correct option is (d)**

75. Under a rotation about the $x$-axis, any vector along the $\hat{x}$-axis will remain invariant.
**Correct option is (c)**

# PAPER : DU 2016
## PHYSICS-PH

**INSTRUCTIONS:**

(*i*) This test booklet contains 45 Objective type questions.

(*ii*) Each question carries **4 marks. 1 mark** will be deducted for each incorrect answer. Only one box should be marked and the marking of more than one box will be treated as wrong answer.

## OBJECTIVE QUESTIONS

1. In a collision between two particles
(a) linear momentum is conserved, but kinetic energy is not necessarily conserved
(b) kinetic energy is conserved, but linear momentum is not necessarily conserved
(c) either linear momentum or kinetic energy, but not necessarily both are conserved.
(d) both linear momentum and kinetic energy are conserved.

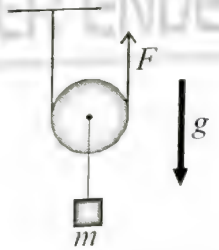
2. The moment of inertia of a thin rectangular plate of length $a$, width $b$, mass $m$ about an axis passing through the centre and perpendicular to the plate is

(a) $\frac{1}{12}m\left(a^2+b^2\right)$ (b) $\frac{1}{6}m\left(a^2+b^2\right)$ (c) $\frac{1}{12}mab$ (d) $\frac{1}{6}mab$

3. A smooth sphere rests on a horizontal plane. A point particle slides without friction down the sphere, starting at the top. If $R$ be the radius of the sphere, the velocity of the particle when it leaves the surface would be given by ($g$ is the acceleration due to gravity)

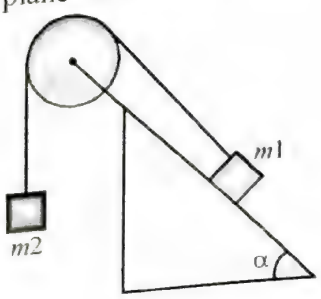
(a) $\sqrt{\frac{2gR}{3}}$ (b) $\sqrt{\frac{3gR}{2}}$ (c) $\sqrt{\frac{3gR}{4}}$ (d) $\sqrt{\frac{4gR}{3}}$

4. A mass $m$ is suspended from a frictionless pulley and hangs over an inextensible string attached to the ceiling on one side and a force $F$ is exerted upwards at the other end as shown in the figure below. The force $F$ for the condition of static equilibrium is




(a) 1 mg (b) 2 mg (c) 4 mg (d) 0.5 mg

5. Two masses $m_1$ and $m_2$ are attached to an inextensible string passing over a massless and frictionless pulley. Mass $m_1$ slides on an inclined plane whereas mass $m_2$ hangs freely as shown in figure below. The acceleration of $m_1$ down the plane would be



(a) $\frac{m_1 - m_2}{m_1 + m_2} g$ (b) $\frac{m_1 \sin\alpha + m_2}{m_1 + m_2} g$ (c) $\frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} g$ (d) $\frac{m_1 \sin\alpha - m_2}{m_1 + m_2} g$

6. Kirchoff's current law is valid for
(a) DC circuit only (b) AC circuit only
(c) Both DC and AC circuits (d) Circuits having active elements only

7. If $\alpha = 0.98$, $I_{CO} = 6\ \mu A$ and $I_B = 100\ \mu A$ for a transistor based amplifier, then value of $I_C$ is about
(a) 2.3 mA (b) 3.1 mA (c) 4.6 mA (d) 5.2 mA

8. The threshold voltage $V_T$ is negative for
(a) an n-channel enhancement MOSFET (b) an n-channel depletion MOSFET
(c) a p-channel depletion MOSFET (d) all active unipolar devices

9. The 2's complement of the binary number 1101100 in BCD is
(a) 12 (b) 13 (c) 14 (d) 15

10. A master-slave flip-flop has the characteristic that
(a) change in the input at 'Master' is immediately reflected in the output of 'Slave'.
(b) input states of both the 'Master' and 'Slave' are affected at the same time.
(c) change in the output of "Slave" follows the change in output of 'Master'.
(d) output states of both the 'Master' and 'Slave' are affected at the same time.

11. Which of the following statements is NOT correct ?
(a) Acousatic branch of diatomic linear chain is similar to the monoatomic case
(b) Both group velocity and phase velocity are equal to the velocity of sound in the long wavelength limit
(c) Under the low wavelength limit, the lattice acts as a low pass filter
(d) Acoustic and optical modes in diatomic lattice cancel each other at the boundary of first Brillouin zone

12. According to Dulong-Petit's law, the specific heat of a solid
(a) is proportional to the temperature (b) does not depend on temperature
(c) depends on square of temperature (d) is inversely proportional to temperature

13. In the X-ray diffraction pattern of a sodium metal, which has a bcc structure, the missing reflection planes will be
(a) 101 (b) 011 (c) 020 (c) 100

14. For a paramagnetic material, the energy difference between spin magnetic dipole parallel and anti-parallel to an external field H is (μB is the Bohr magnetron)
(a) $\mu_0 H / 4\pi$ (b) $\mu_B H$ (c) $2\mu_B \mu_0 H$ (d) $\mu_B \mu_0 H$

15. The Hall coefficient of a Si wafer was found to be $-7.35\times10^{-5}\ m^3 C^{-1}$ in the temperature range from100 to 400 K. The type of charge carrier and the approximate value of carrier density respectively are
(a) n-type; $8.5\times10^{22}\ m^{-3}$ (b) p-type; $8.5\times10^{22}\ m^{-3}$
(c) n-type; $4.2\times10^{22}\ m^{-3}$ (d) p-type; $4.2\times10^{22}\ m^{-3}$

16. Consider oxygen gas at 300K having the mass of its molecule as $5.31\times10^{-26}$ kg. The root mean square speed of its molecules, is about
(a) 284 m/s (b) 248 m/s (c) 348 m/s (d) 484 m/s

17. One mole of a monatomic perfect gas initially at temperature 0°C expands from volume $V_0$ to $2V_0$ at constant pressure. The specific heat at constant volume of the gas is 20.8 $Jmol^{-1}K^{-1}$. The amount of heat absorbed is nearly

(a) 1179 J (b) 1779 J (c) 1979 J (d) 7972 J

18. When 100 g ice at 0°C melts, the change in entropy in cal/K is about (Take latent heat of fusion (melting) as 80 cal/g.)

(a) zero (b) 0.34 (c) 29.3 (d) 39.2

19. N distinguishable particles are distributed among three states having energies 0, $k_BT$ and $2k_BT$ respectively. If the total equilibrium energy of the system is 151.23 $k_BT$, the number of particles of the system is about

(a) 152 (b) 264 (c) 356 (d) 635

20. The power per unit area emitted by a surface of a blackbody (in $Wm^{-2}$) at temperature 230 K is about

(a) 112.67 (b)128.67 (c) 158.67 (d) 178.67

21. A Van der Waals gas is passed through a porous plug. Let $\alpha = \frac{1}{V}\left[\frac{\partial V}{\partial T}\right]_P$ be the volume coefficient and T is the temperature. The temperature of the gas will decrease when

(a) $\alpha T = 0$ (b) $\alpha T > 1$ (c) $\alpha T = 1$ (d) $\alpha T < 1$

22. Light from a point source located at the origin gets reflected parallel to the $x$-axis from a large concave mirror. For a constant, $\alpha$, the equation describing the shape of the mirror on the $x$-$y$ plane is

(a) $y^2 = \alpha x + \alpha^2$ (b) $y^2 = 2\alpha x + \alpha^2$ (c) $x^2 = \alpha y + \alpha^2$ (d) $x^2 = 2\alpha y + \alpha^2$

23. The dispersion relation for a wave is given by $\omega^2 = pk + qk^3$, where $\omega$ is the angular frequency, $k$ is the wave number, $p$ and $q$ are constants. The wave number for which both the group and phase velocities are same is

(a) $\sqrt{\frac{p}{q}}$ (b) $\sqrt{\frac{2p}{q}}$ (c) $\sqrt{\frac{p}{2q}}$ (d) $\frac{1}{2}\sqrt{\frac{2p}{q}}$

24. A thin film of oil of thickness $t$ and refractive index $n_0$ is covering a pool of water of refractive index $n_w$. A ray of light of wavelength $\lambda$ is incident normally on the oil surface. The condition for constructive interference of the reflected light is ($m$ has integer values)

(a) $2tn_w = m\lambda$ (b) $2tn_0 = m\lambda$ (c) $2tn_0 = \left(m + \frac{1}{2}\right)\lambda$ (d) $2tn_w = \left(m + \frac{1}{2}\right)\lambda$

25. A Newton's ring experiment uses a glass lens having radius of curvature 1.0 m. The apparatus is illuminated separately by light having two different wavelengths. Suppose that one of the wavelengths is 550 nm. The 6th bright ring of the 550 nm fringe system coincides with the 5th bright ring of the other. The value of second wavelength is about

(a) 458 nm (b) 672 nm (c) 733 nm (d) 563 nm

26. The position of a particle along the x-axis as a function of time is given by $x(t) = 2\cos^2\left(\frac{\omega_1 t}{2}\right)\sin(\omega_2 t)$. We can write $x(t)$ as the superposition of n independent harmonic motions. The value of $n$ is

(a) 2 (b) 3 (c) 4 (d) 5

27. Two coherent monochromatic light beams of intensities I and 4I are superimposed. The maximum and minimum possible internsities in the resulting beam are

(a) 3I and I (b) 5I and 3I (c) 7I and 5I (d) 9I and I

28. A beam of light polarized in the $x$-direction is travelling in the $z$-direction. It passes through a polarizer and reaches an analyzer. The polarization axis of the analyzer can only be rotated in the $x$-$y$ plane. It is found that the intensity is 3 times smaller when the polarization axis of the analyzer is in the y direction as compare to that in the $x$-direction. The angle between the polarization axis of the polarizer and the x-axis is

(a) 30° (b) 45° (c) 60° (d) 90°

29. Consider a parallel plate capacitor with square plates of dimensions $L \times L$ each. The plates have a charge $Q$ and are separated by a distance $\Delta x$. The plate with the positive charge has a small hole in the middle through which an electron of mass $m$ and charge $-e$ is shot through. The minimum speed, $V$ that the electron must have to reach the negative plate is

(a) $V = \sqrt{\frac{2Qe\Delta x}{m\varepsilon_0 L^2}}$ (b) $V = \sqrt{\frac{Qe\Delta x}{m\varepsilon_0 L^2}}$ (c) $V = \left(\frac{-Qe\Delta x}{m\varepsilon_0 L^2}\right)^2$ (d) $V = \frac{Q^2 e\Delta x}{m\varepsilon_0 L^2}$

30. Two events take place at the same place in a lab frame but occur with a time difference of 3 seconds. The same events occur with a time difference of 5 seconds in a rocket frame. The relative speed of rocket and the laboratory is

(a) 0.5c (b) 0.8c
(c) 0.6c (d) cannot be determined from this data

31. A spaceship approaches a planet with a speed 0.6c. At some point it fires a projectile with a speed 0.4c towards the planet. The speed of the projectile, as seen by an observer on the planet would be approximately

(a) 0.76c (b) 0.80c (c) 0.99c (d) 0.40c

32. The frequency of an LC oscillator is $\omega_0$. The plates of the parallel plate capacitor are pulled apart to twice the original distance, and a dielectric (with dielectric constant $\kappa > 1$) is completely inserted into the capacitor. The new frequency of oscillation for the circuit is

(a) $\sqrt{\frac{2}{\kappa}}\omega_0$ (b) $\frac{2}{\kappa}\omega_0$ (c) $\sqrt{\frac{\kappa}{2}}\omega_0$ (d) $\frac{\kappa}{2}\omega_0$

33. A particle is moving at a speed of $2.6\times10^8\ ms^{-1}$ relative to the laboratory. Its lifetime as measured by an observer in the laboratory is $4.7\times10^{-6}$ s. The lifetime of the particle in its own rest frame is about

(a) $2.3\times10^{-6}s$ (b) $9.4\times10^{-6}s$ (c) $4.7\times10^{-6}s$ (d) $14.4\times10^{2}s$

34. For a wave function defined as $\psi(x)=\left(\frac{2}{L}\right)^{1/2}\sin\left(\frac{\pi x}{L}\right)$ in the region $0<x<L$ and $\psi(x)=0$ outside this region, the average value of the square of the momentum $\left(p^2\right)$ is,

(a) $\frac{\hbar\pi}{L}$ (b) $\frac{h^2\pi^2}{L^2}$ (c) $\frac{\hbar^2\pi^2}{L^2}$ (d) 0

35. A sample of radioactive $^{235}$Pa nucleus undergoes beta decay with a half-life $\left(t_{1/2}\right)$ of 24 minutes. If the activity of this radioactive sample is 1 Curie, its mass will be about

(a) $3.0\times10^{-8}\,gm$ (b) $8.2\times10^{-9}\,gm$ (c) $4.9\times10^{-9}\,gm$ (d) $7.5\times10^{-8}\,gm$

36. The ground state spin and parity of $^{19}F_9$ nuclei will be –

(a) Proton : $J^{\pi}=\left(\frac{5}{2}\right)^{+}$ (b) Neutron : $J^{\pi}=\left(\frac{5}{2}\right)^{-}$

(c) Proton : $J^{\pi}=\left(\frac{3}{2}\right)^{+}$ (d) Neutron : $J^{\pi}=\left(\frac{3}{2}\right)^{-}$

37. The kinetic energy of the electrons emitted from the n = 3 state of the hydrogen atom, when illuminated with blue light of wavelength 450 nm, is approximately

(a) 2.76 eV (b) 4.27 eV (c) 1.51 eV (d) 1.25 eV

38. The binding energies of $^1$H, $^4$He and $^7$Li are $B_1$, $B_4$ and $B_7$ respectively. The Q value of the reaction $^1H+{}^7Li\rightarrow 2\,{}^4He+Q$ is given by

(a) $2B_1+4B_7-6B_4$ (b) $B_1+B_7-2B_4$ (c) $B_4-B_1-B_7$ (d) $B_1+7B_7-8B_4$

39. Consider the matrix

$$A=\begin{pmatrix}2 & 2\\ 2 & 5\end{pmatrix}$$

If $B = 2e^A$, the determinant of B is

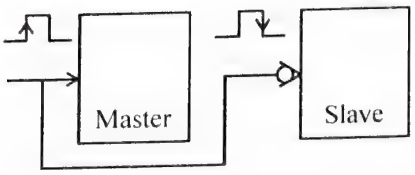
(a) $4e^7$ (b) $4\left(e^7-e^4\right)$ (c) $e^7$ (d) $e^7-e^4$

40. The value of the determinant $\begin{vmatrix}2 & 5 & 3+2i\\ 9 & 2 & 5-4i\\ -20 & -9 & 3+6i\end{vmatrix}$ is

(a) 656 + 256i (b) –656–256i (c) 656 (d) –656

41. The most general solution of the differential equation $\frac{d^2y}{dx^2}+\frac{4dy}{dx}+4y=0$ is (here 'a' and 'b' are constants)

(a) $ae^{-2x}+be^{2x}$ (b) $ae^{-2x}+be^{-2x}$ (c) $ae^{-2x}+bxe^{-2x}$ (d) $ae^{2x}+bxe^{2x}$

10. 


Change in state of slave follow's as per change in master state.
**Correct option is (c)**

11. **Correct option is (d)**

12. According to Dulong-Petit's law, the specific heat of any solid is given by $3R$.
**Correct option is (b)**

13. Na has BCC structure, so diffraction will be observed, if the $h+k+l=$ even .
Only option (d) will not satisfy this condition.
**Correct option is (d)**

14. The $p$ magnetostatic energy of spin which keep in external magnetic field.

$u=-\vec{\mu}\cdot\vec{B}$ ($\mu$ spin magnetic moment)

$=-\mu_B\,\mu_0 H$ (Spin parallel to $\vec{H}$)

$=\mu_B\,\mu_0 H$ (Spin anti-parallel to $\vec{H}$)

Therefore, the energy difference $=\mu_B\,\mu_0 H-(-\mu_B\,\mu_0 H)=2\mu_B\,\mu_0 H$

**Correct option is (c)**

15. The Hall coefficient is given by $R_H=\pm\dfrac{1}{ne}$

Positive for p-type semiconductor.
Negative for n-type semiconductor.
The given semiconductor is $n$-type, so charge carries will be $n$-type.

Now, $\dfrac{1}{ne}=7.35\times10^{-3}$

$\Rightarrow\ n=\dfrac{1}{7.35\times10^{-5}\times1.6\times10^{-19}}/\text{m}^3$

$=8.5\times10^{22}\ \text{m}^{-3}$

**Correct option is (a)**

16. $v_{\text{r.m.s.}}=\sqrt{\dfrac{3k_BT}{m}}=\sqrt{\dfrac{3\times1.38\times10^{-23}\times300}{5.31\times10^{-26}}}\simeq484\ \text{m/sec}$

**Correct option is (d)**

17. At constant pressure, we have $\dfrac{V}{T}=\text{constant}$

$\Rightarrow\ T_f=\dfrac{V_f}{V_i}T_i=\dfrac{2V_0}{V_0}\times273=546\ \text{K}$

$\therefore \Delta T = T_f - T_i = 546 - 273 = 273$ K

$\therefore \Delta Q = nC'_P \Delta T$

$= n(C'_V + R)\Delta T$

$= (20.8 + 8.3) \times 273$

$= 7944.3$ J

$\simeq 7972$ (nearest integer)

**Correct option is (d)**

18. The change in entropy is

$$\Delta S = \frac{mL}{T} = \frac{100 \times 80}{273} = 29.3 \text{ cal/K}$$

**Correct option is (c)**

19. The average energy for $N$ distinguishable particles is

$$\langle E \rangle = N \frac{\sum_i E_i g_i e^{-E_i/k_B T}}{\sum_i g_i e^{-E_i/k_B T}}$$

$$\Rightarrow 151.23\, k_B T = N \left| \frac{k_B T e^{-1} + 2k_B T e^{-2}}{1 + e^{-1} + e^{-2}} \right|$$

$$\Rightarrow N = 151.23 \left( \frac{e^2 + e + 1}{e + 2} \right) \simeq 356$$

**Correct option is (c)**

20. The power $P$ emitted by a blackbody of area $A$ at temperature $T$ is

$$P = \sigma A T^4 \qquad \left(\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4\right)$$

$$\Rightarrow \frac{P}{A} = \sigma T^4 = 5.67 \times 10^{-8} \times (230)^4$$

$$\Rightarrow \frac{P}{A} = 158.67 \text{ W/m}^2$$

**Correct option is (c)**

21. The Joule-Thomson coefficient is

$$\mu_{JT} = \frac{V}{C_P}(\alpha T - 1)$$

The gas will cool down when $\mu_{JT} > 0$

$\alpha T - 1 > 0$

$\Rightarrow \alpha T > 1$

**Correct option is (b)**

22. According to Fermat principal, light travel along the path for which the time or path is stationary, least, or extremum.

Here, rays reflected from the mirror will have stationary path, thus

$FPN = FOF$

$FP + PN = 2(FM + MO)$

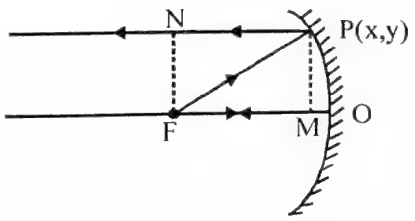
$\sqrt{x^2 + y^2} + x = 2[x + MO]$

$\Rightarrow \sqrt{x^2 + y^2} = x + 2MO$

$x^2 + y^2 = x^2 + 2.x.MO + 4MO^2$

$y^2 = 2\alpha x + \alpha^2$

where $\alpha = 2MO$ is a constant.




**Correct option is (b)**

23. We have $\omega^2 = pk + qk^3$

$\Rightarrow \omega = \sqrt{pk + qk^3}$

$\therefore$ Phase velocity $v_p = \dfrac{\omega}{k} = \dfrac{1}{k}\sqrt{pk + qk^3}$

And group velocity $v_y = \dfrac{d\omega}{dk} = \dfrac{1}{2}\dfrac{p + 3qk^2}{\sqrt{pk + qk^3}}$

Given : $v_g = v_p$

$\Rightarrow \dfrac{1}{2}\dfrac{p + 3qk^2}{\sqrt{pk + qk^3}} = \dfrac{1}{k}\sqrt{pk + qk^3} \quad \Rightarrow pk + 3qk^3 = 2\left(pk + qk^3\right)$

$\Rightarrow pk = qk^3 \quad \Rightarrow k = \sqrt{\dfrac{p}{q}}$

**Correct option is (a)**

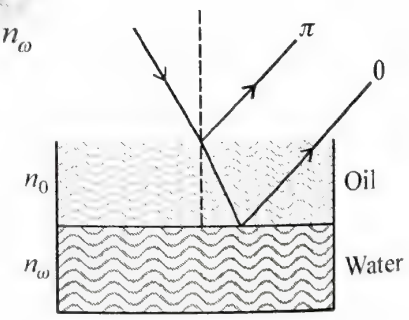
CAREER ENDEAVOUR

24. In general refractive index of oil is greater than the water, i.e., $n_0 > n_\omega$

Here, additional phase change is $\delta_{add} = \pi - 0 = \pi$ .

Since, oil have thin layer, path difference for maxima is

$2t\,n_0 = (2m+1)\dfrac{\lambda}{2} = \left(m + \dfrac{1}{2}\right)\lambda$




**Correct option is (c)**

25. Position of bright ring is

$r_m^2 = \left(m + \dfrac{1}{2}\right)\lambda R;\ m = 0, 1, 2, 3, \ldots.$

Position of 6th bright ring is for $m = 5$

$r_6^2 = \left(5 + \dfrac{1}{2}\right)\lambda_1 R$

Position of 5th bright ring is for $m = 4$ ; $r_5^2 = \left(4 + \dfrac{1}{2}\right)\lambda_2 R$

As $r_6 = r_5$

$$\Rightarrow \left(5+\frac{1}{2}\right)\lambda_1 R = \left(4+\frac{1}{2}\right)\lambda_2 R$$

$$\Rightarrow 5.5\lambda_1 = 4.5\lambda_2$$

$$\Rightarrow \lambda_2 = \frac{5.5}{4.5}\lambda_1 = \frac{5.5}{4.5} \times 550 \text{ nm} = 672 \text{ nm}$$

**Correct option is (b)**

26. Displacement of the particle

$$x(t) = 2\cos^2\frac{\omega_1 t}{2}\sin\omega_2 t$$

$$= [1+\cos\omega_1 t]\sin\omega_2 t$$

$$= \sin\omega_2 t + \cos\omega_1 t \sin\omega_2 t$$

$$= \sin\omega_2 t + \frac{1}{2}\left[\sin\left(\frac{\omega_1+\omega_2}{2}\right)t + \sin\left(\frac{\omega_2-\omega_1}{2}\right)t\right]$$

Thus, $x(t)$ is superposition of three independent harmonic oscillator.

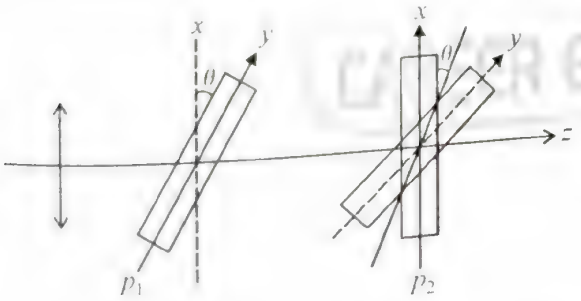
**Correct option is (b)**

27. Intensity of maxima $I_{max} = \left[\sqrt{I_1}+\sqrt{I_2}\right]^2 = \left[\sqrt{4I}+\sqrt{I}\right]^2 = 9I$

Intensity of minima $I_{min} = \left[\sqrt{I_1}-\sqrt{I_2}\right]^2 = \left[\sqrt{4I}-\sqrt{I}\right]^2 = I$

**Correct option is (d)**

28.




Let $\theta$ be the angle between $x$-axis and polarizer $p_1$.

Intensity after $p_1$ is $I_1 = I_0\cos^2\theta$

When analyzer is along $x$-axis: ... (1)

Intensity after $p_2$ is $I_{2,x} = I_1\cos^2\theta = I_0\cos^2\theta\cdot\cos^2\theta$

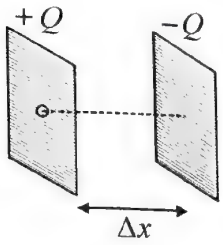
When analyzer is along $y$-axis: ... (2)

Intensity after $p_2$ is $I_{2,y} = I_1\cos^2(90°-\theta) = I_0\cos^2\theta\cdot\sin^2\theta$

Given: $\frac{I_{2,y}}{I_{1,x}} = \frac{1}{3} \Rightarrow \frac{I_0\cos^2\theta\sin^2\theta}{I_0\cos^2\theta\cdot\cos^2\theta} = \frac{1}{3} \Rightarrow \tan^2\theta = \frac{1}{3} \Rightarrow \theta = 30°$

**Correct option is (a)**

29. The electric field between two plates is




$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{L^2\varepsilon_0}$$

So the potential difference between two plate is

$$V = \left(\frac{Q\Delta x}{L^2\varepsilon_0}\right)$$

Now, we know that, work done = change of kinetic energy

$$\Rightarrow \frac{Q\Delta xe}{L^2\varepsilon_0} = \frac{1}{2}mv^2 - 0 \Rightarrow v^2 = \frac{2Q\Delta xe}{mL^2\varepsilon_0} \Rightarrow v = \sqrt{\frac{2Q\Delta xe}{mL^2\varepsilon_0}}$$

**Correct option is (a)**

30. Let the laboratory be the $S$ frame and the rocket moving with speed $v$ be the $S'$ frame.

In $S$ frame : $\Delta x = 0$, $\Delta t = 3$

In $S'$ frame : $\Delta t' = 5$

Using Lorentz transformation,

$$\Delta t' = \left(\Delta t - \frac{v\Delta x}{c^2}\right)\gamma; \text{ where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow 5 = \left(3 - \frac{v\times 0}{c^2}\right)\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 1 - \frac{v^2}{c^2} = \left(\frac{3}{5}\right)^2$$

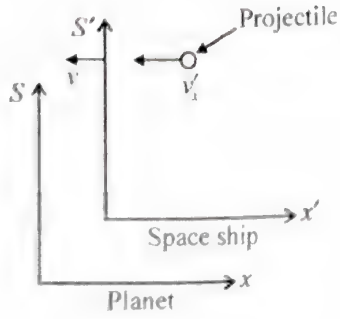
$$\Rightarrow \frac{v^2}{c^2} = \frac{16}{25} \Rightarrow v = \frac{4}{5}c = 0.8c$$

**Correct option is (b)**

31. Applying inverse Lorentz transformation for velocity,

$$v_x = \frac{v_x' + v}{1 + v_x'\frac{v}{c^2}}$$

Putting the values of $v_x' = -0.4c$, $v = -0.6c$, we get




$$v_x = \frac{-0.4c - 0.6c}{1 + \frac{(-0.4c)(-0.6c)}{c^2}}$$

$$v_x = \frac{c}{1+0.24} = -\frac{c}{1.24} \approx -0.80c$$

**Note:** The negative sign gives the direction of velocity.

**Correct option is (b)**

32. The frequency of $LC$ circuit is given by $\omega_0 = \dfrac{1}{\sqrt{LC}}$

The capacitance of parallel plate capacitor without dielectric $C = \dfrac{\varepsilon_0 \cdot A}{d}$

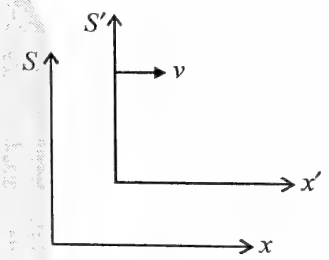
The capacitance of parallel plate capacitor with dielectric $C' = \left(\dfrac{\varepsilon_0 \kappa A}{2d}\right) = \dfrac{\kappa C}{2}$

Therefore, the frequency of the $LC$ circuit will be

$$\omega' = \frac{1}{\sqrt{LC'}} = \frac{1}{\sqrt{\frac{\kappa}{2}LC}} = \sqrt{\frac{2}{\kappa}}\,\omega_0$$

**Correct option is (a)**

33. Let laboratory be the $S$ frame, and the particle be the $S'$ frame




$$\tau_{\text{particle}} = \tau_{\text{lab}}\sqrt{1-\frac{v^2}{c^2}}$$

Putting the values of $v = 2.6\times10^8$ ms$^{-1}$, $\tau_{\text{lab}} = 4.7\times10^{-6}$ s, we get

$$\tau_{\text{particle}} = 4.7\times10^{-6} \times \sqrt{1-\left(\frac{2.6\times10^8}{3\times10^8}\right)^2}$$

$$\approx 2.3\times10^{-6}\ \text{s}$$

**Correct option is (a)**

34. The given function is ground state of 1-D box with energy $\dfrac{\pi^2\hbar^2}{2mL^2}$

$$\frac{\langle p^2\rangle}{2m} = \frac{\pi^2\hbar^2}{2mL^2}$$

$$\Rightarrow \langle p^2\rangle = \frac{\pi^2\hbar^2}{L^2}$$

**Correct option is (c)**

35. Activity of the radioactive substance is

$R = -\dfrac{dN}{dt} = -\lambda N$, where $\lambda$ : decay constant, $N$ : number of atom present at that time.

Also, $T_{1/2} = \dfrac{\ln 2}{\lambda} \Rightarrow \lambda = \dfrac{\ln 2}{T_{1/2}} \Rightarrow R = -\dfrac{dN}{dt} = \lambda N = \dfrac{\ln 2}{T_{1/2}}N \Rightarrow N = \dfrac{RT_{1/2}}{\ln 2}$ ... (1)

Given $R = 1$ curie $= 3.7\times10^{10}$ disintegration/sec.

Number of mole 235 Pa in a given substance is $n = \dfrac{N}{N_A} = \dfrac{RT_{1/2}}{\ln 2\, N_A}$

If $M_0$ be the molecular mass, then mass of the substance,

$$m = M_0 n = \frac{235\,gm \times 3.7 \times 10^{10} \times 24 \times 60}{0.693 \times 6.022 \times 10^{23}} = 3.0 \times 10^{-8}\ gm$$

**Correct option is (a)**

36. $^{19}F_9 : Z = 9\,(\text{odd}),\ N = 10\,(\text{even})$

$Z = (^1s_{1/2})^2\ (^1p_{1/2})^2\ (^1p_{3/2})^4\ (^1d_{5/2})^1$

Spin of the nucleus = 5/2

Parity $p = (-1)^l = (-1)^2 = +1 \Rightarrow$ Spin and parity $= \frac{5^+}{2}$

**Correct option is (a)**

37. Energy of the $n^{\text{th}}$ state of a hydrogen atom $(E_n) = -\frac{13.6}{n^2}\,\text{eV}$

$\therefore$ For $n = 3$, $E_3 = -\frac{13.6}{3^2} = -1.51\,\text{eV}$

The energy of a photon of wavelength $\lambda$ is $(E_{\text{photon}}) = \frac{1241\,\text{eV-nm}}{\lambda}$

Here, $\lambda = 450$ nm

$\therefore$ $E_{\text{photon}} = \frac{1241\,\text{eV-nm}}{450\,\text{nm}} = 2.76\,\text{eV}$

$\because$ The emitted electron has become free, its total energy will be equal to kinetic energy. (Since, potential energy = 0).

$\therefore$ Kinetic energy = (–1.51 to 2.76) eV = 1.25 eV.

**Correct option is (d)**

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38. Reaction given is $^1_1\text{H} + ^7_3\text{Li} \longrightarrow 2\,^4_2\text{He} + \text{Q}$

The Q-value of above reaction is given by

Q = Sum of mass energy of reactants – sum of mass energy of the produced

= Sum of binding energy of reactants – Sum of binding energy of the product

$$Q = (B_1 + B_7) - (B_4 + B_4) \Rightarrow Q = B_1 + B_7 - 2B_4$$

**Correct option is (b)**

39. $A = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix}$

We know that, $\det(e^A) = e^{\text{Trace}(A)}$ and $\det(\text{mA}) = m^n \det(A)$, where $n$ is the order of $A$

$\det B = \det(2e^A) = 2^2 \det(e^A) = 4e^{\text{Tr}(A)} = 4e^{2+5} = 4e^7$

**Correct option is (a)**

40. $\begin{vmatrix} 2 & 5 & 3+2i \\ 9 & 2 & 5-4i \\ -20 & -9 & 3+6i \end{vmatrix} = 2[6+12i+45-36i]+5[-100+80i-27-54i]+(3+2i)[-81+40]$

$= 2(51-24i)+5(-127+26i)-41(3+2i)$

$= (102-635-123)-48i+130i-82i$

$= -656$

**Correct option is (d)**

41. $\frac{d^2y}{dx^2}+\frac{4dy}{dx}+4y=0$

The auxilliary equation is $m^2+4m+4=0$

$\Rightarrow (m+2)^2=0 \Rightarrow m=-2,-2$

$\therefore\ y(x)=Ae^{-2x}+Bxe^{-2x}$

**Correct option is (c)**

42. $\int_1^\infty e^{-(x-1)^3}(x-1)^5\,dx = \int_1^\infty e^{-(x-1)^3}(x-1)^3(x-1)^2\,dx$

Taking $(x-1)^3=t \Rightarrow 3(x-1)^2dx=dt$

$= \frac{1}{3}\int_0^\infty te^{-t}dt = \frac{1}{3}\Gamma(2)=\frac{1}{3}$

**Correct option is (c)**

43. Basis of a 2-dimensional vector space can be formed from any two vectors in the 2-D plane which are linearly independent of each other.
Here, all the sets of vectors are set of linearly independent vectors.
**Correct option is (a)**

44. $a^2x^2+y^2=2(x+yb)$

$\Rightarrow a^2x^2-2x+y^2-2by=0$

$\Rightarrow a^2\left(x^2-\frac{2}{a^2}x\right)+\left(y^2-2by\right)=0$

$\Rightarrow a^2\left[\left(x-\frac{1}{a}\right)^2-\frac{1}{a^2}\right]+\left[(y-b)^2-b^2\right]=0$

or $\left(x-\frac{1}{a}\right)^2+\frac{(y-b)^2}{a^2}-\frac{1}{a^2}-\frac{b^2}{a^2}=0$

or $\left(x-\frac{1}{a}\right)^2+\frac{(y-b)^2}{a^2}=\frac{1+b^2}{a^2}$

This is the equation of an ellipse.
**Correct option is (d)**

45. Initial angular momentum $= I\omega = \frac{1}{2}MR^2\omega$

Moment of inertia of the composite disc

$$= \frac{1}{2}MR^2 + \frac{1}{2}M\left(\frac{R}{2}\right)^2$$

$$= \left(\frac{1}{2} + \frac{1}{8}\right)MR^2$$

$$= \frac{5}{8}MR^2$$

Let the final angular velocity of the composite disc be $\omega'$.

Final angular momentum $= I'\omega' = \frac{5}{8}MR^2\omega'$

Using conservation of angular momentum, $I\omega = I'\omega'$

$$\Rightarrow \frac{1}{2}MR^2\omega = \frac{5}{8}MR^2\omega' \Rightarrow \omega' = \frac{4}{5}\omega$$

**Correct option is (b)**

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# PAPER : DU 2017
## PHYSICS-PH

**INSTRUCTIONS:**

(i) This test booklet contains 50 Objective type questions.

(ii) Each question carries **4 marks**. **1 mark** will be deducted for each incorrect answer. Only one box should be marked and the marking of more than one box will be treated as wrong answer.

## OBJECTIVE QUESTIONS

1. Given the position vector $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ and $r^2 = \vec{r}\cdot\vec{r}$, the value of $\vec{\nabla}\cdot(r^2\vec{r})$ is given by :

(a) $6r^2$ (b) $5r^2$ (c) $4r^2$ (d) $3r^2$

2. The solution to the differential equation $\frac{dy}{dx} + 2xy = x$ given that $y(x=0) = 0$ is

(a) $y = (1 - e^{-x^3})$ (b) $y = \frac{1}{4}(1 - e^{-x^2})$ (c) $y = \frac{1}{2}(1 - e^{-x^2})$ (d) $y = \frac{1}{2}(1 - e^{-x})$

3. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and I is a 2 × 2 identity matrix, then $e^A$ is :

(a) $I + \left(\frac{e^2 - 1}{2}\right)A$ (b) $I + \left(\frac{e - 1}{2}\right)A$ (c) $I + \left(\frac{e^2 + 1}{2}\right)A$ (d) $I + \left(\frac{e + 1}{2}\right)A$

4. If $z = x + iy$, then the real part of cos $(z)$ is :

(a) $\cos(x)\cosh(y)$ (b) $\cos(x)\cos(y)$ (c) $\sin(x)\sinh(y)$ (d) $-\sin(x)\sinh(y)$

5. The heat extracted in each cycle from its heat source of a Carnot engine having an efficiency of 59% and performing $2.5 \times 10^4$ J of work in each cycle is :

(a) $4.24 \times 10^2$ J (b) $2.12 \times 10^2$ J (c) $4.24 \times 10^4$ J (d) $2.12 \times 10^4$ J

6. The probability that an allowed state is occupied if it lies above the Fermi level by $2\,k_B T$ is

(a) 24 % (b) 12 % (c) 2.4 % (d) 1.2 %

7. One mole of a monatomic perfect gas is initially at a constant temperature of $T_0$. It expands from a volume $V_0$ to $2V_0$ under constant pressure. The heat absorbed by the gas is

(a) $RT_0$ (b) $2RT_0$ (c) $\frac{3}{2}RT_0$ (d) $\frac{5}{2}RT_0$

8. To a pure substance undergoing an infinitesimal reversible process, which of the following relation is not a Maxwell equation ?

(a) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ (b) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial V}{\partial S}\right)_P$

(c) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ (d) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$

9. A self-contained machine has, as its input, two equal steady streams of a fluid at temperatures $T_1$ and $4T_1$, respectively. The only output is a single high-speed jet of the same fluid at temperature $T$. The heat capacity per unit mass of the fluid is $C$ (and is independent of the temperature). What is the maximum possible speed of the jet ?
(Hint : The process is irreversible)

(a) $\sqrt{\frac{CT_1}{4}}$ (b) $\sqrt{4CT_1}$ (c) $\sqrt{2CT_1}$ (d) $\sqrt{CT_1}$

10. A freezer in a fridge is maintained at a temperature of –15 °C where the ambient air temperature is 30 °C. If heat leaks into the freezer at the constant rate of 1.75 kJ/sec, then the power needed to pump this heat out continuously is
(a) – 0.13 kJ/sec (b) 0.31 kJ/sec (c) 1.49 kJ/sec (d) 2.06 kJ/sec

11. Consider a random walker in one dimension. Assume that at every time step the walker takes steps of unit length to the left or right. The probability of taking a step to the left is 1/3 and to the right is 2/3. If the walker starts at $x = 0$, then the probability of its return to its starting point after four steps is given by

(a) $\frac{2^2}{3^2}$ (b) $\frac{2^2}{3^3}$ (c) $\frac{2}{3^3}$ (d) $\frac{1}{3^3}$

12. Consider a system of 3 non-interacting identical bosons. Each particle can be in any one of three states whose energies are $-\varepsilon, 0, +\varepsilon$. Find the entropy of the system if its total energy $E = 0$.
(a) 0 (b) $k \ln 8$ (c) $k \ln 4$ (d) $k \ln 2$

13. A rectangular coil of length 0.05 m and breadth 0.08 m and having 420 turns is placed horizontally in a uniform magnetic field $B_1 = 0.14\,T$ going into the plane of the coil. The magnetic field is increased over a 0.20 second interval and this induces an emf of 1.8 V in the coil. The final magnetic field $B_2$ is approximately :
(a) 0.80 T (b) 0.35 T (c) 0.14 T
(d) Cannot be determined from the information supplied

14. Consider a plane electromagnetic wave travelling in free space. The magnetic field is given by $\vec{B} = B_1(\hat{x} - \hat{z})$ and electric field is given by $\vec{E} = E_1(-\hat{x} + 2\hat{y} - \hat{z})$. Then the direction of the propagation of the wave is
(a) $-\hat{x} - \hat{y} - \hat{z}$ (b) $\hat{x} + 2\hat{y} - 2\hat{z}$ (c) $2\hat{x} + \hat{y} + 2\hat{z}$ (d) None of these

15. A particle of mass $m$ and charge $q$ is released from rest in a region with $\vec{E} = E_0\hat{x}$ and $\vec{B} = B_0\hat{x}$, where $E_0, B_0$ are constants. The particle's motion is best described by which of the following ?

(a) It moves in a circle with a speed, $v = \frac{qE_0 t}{m}$

(b) It moves along the $\hat{x}$ axis with an acceleration $\frac{qE_0}{m}$

(c) It moves in a helical path in the $xz$-plane with an increasing radius

(d) It moves along the $\hat{z}$ direction with an acceleration $\frac{qvE_0}{m}$

16. A sphere of radius $a$ has a charge density which varies with distance as $\rho = A\sqrt{r}$. The electric field at a distance $r < a$ varies with distance $r$ as :

(a) $E \propto r^{3/2}$ (b) $E \propto r^{1/2}$ (c) $E \propto r^{-1/2}$ (d) $E \propto r^{-2}$

17. A long cylindrical conductor kept along the $z$-axis carries a current density $\vec{J} = j_0 r\hat{z}$, where $j_0$ is a constant, and $r$ is the radial distance from the axis of the cylinder. The magnetic induction $\vec{B}$ inside the conductor at a distance $d$ from the axis of the cylinder is :

(a) $\mu_0 \hat{J}_0 \hat{\phi}$ (b) $-\frac{1}{2}\mu_0 \hat{J}_0 d\hat{\phi}$ (c) $\frac{1}{3}\mu_0 j_0 d^2 \hat{\phi}$ (d) $-\frac{1}{4}\mu_0 j_0 d^3 \hat{\phi}$

18. Which of the following statements is true for a series LCR circuit with an AC source (where R is a resistance, L is an inductor and C is a capacitor) :

(a) The impedance offered by the capacitor decreases as the frequency of the AC voltage increases
(b) The voltage and current are always in phase
(c) The total impedance of the circuit will be less than R
(d) The total impedance of the circuit approaches R as the frequency become very small

19. What is the direction of the displacement current inside a discharging cylindrical capacitor having circular positive and negative plates parallel to reach other ?

(a) From positive plate to the negative plate
(b) Directed towards the axis of the capacitor
(c) Directed away from the axis of the capacitor
(d) From the negative plate to the positive plate

20. Two dice are thrown together. Let S be the sum of the two outcomes. Which of the following statements is false?

(a) The probability that S = 1 is 0 (b) The probability that S = 6 is 1/6
(c) The probability that S = 2 is 1/36 (d) The probability that S = 3 is 1/18

21. The normalized wave functions $\Psi_1$ and $\Psi_2$ correspond to the ground state and the first excited states of a particle in a potential. The operator $\hat{A}$ acts on the wavefunctions as $\hat{A}\Psi_1 = \Psi_2$ and $\hat{A}\Psi_2 = \Psi_1$. The expectation value of the operator $\hat{A}$ for the state $\hat{A} = (3\Psi_1 + 4\Psi_2)/5$ is

(a) 0 (b) – 0.32 (c) 0.75 (d) 0.96

22. The de-Broglie wavelength of a particle of mass $m$ and charge $q$ after being accelerated through a potential difference of V volts is

(a) $\lambda = \frac{h}{\sqrt{2mqV}}$ (b) $\lambda = \frac{h}{\sqrt{mqV}}$ (c) $\lambda = \frac{hm}{\sqrt{qV}}$ (d) $\lambda = \frac{h}{qmV}$

23. The state of a hydrogen atom is described by the wavefunction $\Psi = Ar^2 \exp\left(-\frac{r}{3a_0}\right)(3\cos^2\theta - 1)$, where A and $a_0$ are the normalization constant and Bohr radius, respectively. If one measures the $z$-component of the orbital angular momentum for this atom, the outcomes will be :

(a) $3\hbar$ since $\Psi$ is proportional to $\exp\left(-\frac{r}{3a_0}\right)$

(b) $2\hbar$ since $\Psi$ depends on $\cos^2\theta$

(c) $1\hbar$ since $\Psi$ depends on $\cos^2\theta$

(d) $0\hbar$ since $\Psi$ does not depend on $\phi$

24. An electron is confined within a region of size $< 10^{-15}$ m by an external potential. Then, the average kinetic energy of this electron must be
(a) much greater than 0.511 MeV (b) approximately equal to 0.511 MeV
(c) approximately equal to $0.511 \times 10^{-13}$ eV (d) approximately equal to 1 eV

25. An amplifier with initial open loop gain of 400 is used as negative feedback amplifier. The feedback factor is 0.05. If the gain of the amplifier changes by 10% due to temperature, then the closed loop gain changes approximately by
(a) 0.05 % (b) 0.1 % (c) 0.5 % (d) 1 %

26. What is the value of the least significant bit of an 8-bit Digital to Analog Converter (DAC) for 0 – 12.8 V range?
(a) 0.050 V (b) 0.625 V (c) 1.28 V (d) 1.6 V

27. If the distance between two adjacent peaks on the $x$-axis is 2 cm on a CRO screen in which the time/division control is set at 1 ms/div, then the frequency of the signal is
(a) 1 kHz (b) 5 kHz (c) 50 Hz (d) 500 Hz

28. The decimal equivalent of the hexadecimal number $(A3B)_{16}$ is
(a) $(2361)_{10}$ (b) $(2621)_{10}$ (c) $(2619)_{10}$ (d) $(2879)_{10}$

29. Simplification of the following Boolean expression $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$ is given by
(a) $\bar{A}\bar{C} + AC$ (b) $\bar{A}\bar{C}$ (c) C (d) $\bar{C}$

30. An astronaut in a spaceship has a mass 75 kg in the rest frame of the spaceship. The spaceship's length in its rest frame is 1150 m while its length as observed from the earth is 325 m. The total energy of the astronaut in the earth's frame will be approximately :
(a) 75 J (b) 750 J (c) $2.4 \times 10^{19}$ J (d) $6.8 \times 10^{18}$ J

31. A thick steel rod rotates with an initial angular velocity $\omega$ about one of its endpoints in a horizontal plane. If at time $t = 0$ an ant having mass $m$ starts moving outwards from the point of rotation with constant speed then for the system after time $t > 0$ :
(a) both the angular velocity and the kinetic energy decreases
(b) the angular velocity decreases but kinetic energy increases
(c) both the angular velocity and the kinetic energy remains same
(d) the angular velocity increases but kinetic energy decreases

32. A tunnel is dug from the surface of the earth through the center and opens at the other end. A ball is dropped from one end of the tunnel. The acceleration due to gravity on the earth's surface is $g$ and the radius of the earth is $R$. Assuming that the earth has a constant density, the time taken by the ball to reach the center of the earth is
(a) $\pi\sqrt{\frac{R}{g}}$ (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $\frac{\pi}{2}\sqrt{\frac{R}{g}}$ (d) $\frac{\pi}{4}\sqrt{\frac{R}{g}}$

33. A block of mass 250 g is given an initial speed of 3m/s up a plane inclined at an angle of 30° with the horizontal. The coefficient of friction is $\sqrt{3}/6$. After 1.5 s, how far is the block from its original position ?
(a) 0.9 m (b) 2.4 m (c) 3.8 m (d) 6.5 m

34. The Sun is approximately 25,000 light years away from the centre of the Milky Way and moves around it, in an approximately circular path, in roughly 170 million years. Let M be the mass of the galactic matter within a radius of 25,000 light years from the galactic centre. Given that sunlight takes approximately 8 min to reach the Earth, what is the ratio of M to the mass of the Sun ?

(a) $1.5\times10^{11}$ (b) $5\times10^{11}$ (c) $1.5\times10^{12}$ (d) $5\times10^{12}$

35. Two particles of mass $m$ each are connected by a massless spring of length $l$ and spring constant $k$. The system is lying (in the relaxed state) on a friction horizontal table. A very short impulse P is applied to the first mass (along the axis of the system) at $t=0$. When would the second mass first come to rest ?

(a) $t=\pi\sqrt{\frac{2m}{k}}$ (b) $t=2\pi\sqrt{\frac{2m}{k}}$ (c) $t=\pi\sqrt{\frac{m}{2k}}$ (d) $t=\pi\sqrt{\frac{m}{k}}$

36. A narrow light beam of wavelength $\lambda$ is travelling along the $x$-axis, when it is faced by a barrier with narrow slits at $y=0,\pm d$ (and all at $x=0$). There is a vertical screen at $x=D\gg d$. What is the angle $\theta$ that the central bright spot and the next-nearest dark spot subtend at the central slit $[(x,y)=(0,0)]$ ?

(a) $\theta=\sin^{-1}\frac{\lambda}{3d}$ (b) $\theta=\sin^{-1}\frac{2\lambda}{d}$ (c) $\theta=\sin^{-1}\frac{\lambda}{2d}$ (d) $\theta=\sin^{-1}\frac{\lambda}{d}$

37. Two tuning forks with natural frequencies $\nu_1$ and $\nu_2$, respectively, are struck at the same time with equal force. The intensity of the resulting sound, beats with a frequency of 1 Hz while the frequency of the sound is 512 Hz. Hence, $\nu_1$ and $\nu_2$, respectively, are

(a) 512.0 Hz and 511.5 Hz (b) 512.0 Hz and 511.0 Hz
(c) 512.5 Hz and 511.0 Hz (d) 512.5 Hz and 511.5 Hz

38. During normal to superconducting transition :
(a) Entropy increases and thermal conductivity decreases
(b) Entropy and thermal conductivity decrease
(c) Entropy and thermal conductivity increase
(d) Entropy decreases and thermal conductivity increases

39. A silicon wafer of thickness 200 μm is doped with $10^{23}$ phosphorus atoms per $m^3$. If the sample is kept in a field of 0.2 Weber/$m^2$ and a current of 1 mA is passed through the sample, the Hall voltage produced is

(a) $-6.25$ μV (b) 6.25 μV (c) $-62.5$ μV (d) 62.5 μV

40. In the Kronig-Penney model, the discontinuous in the $E$ vs $k$ curve occur on either side of the centre for a $k$-value equal to (where $n = 1, 2, 3, ....$)

(a) $n\pi/a$ (b) $n\pi/2a$ (c) $(n\pi/2a)^{1/2}$ (d) $2n\pi/a$

41. The fraction of volume occupied by the atoms (hard spheres) in a simple cubic unit cell of lattice parameter $a$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

42. The energy gap of Indium (In) which starts superconducting at 3.4 K is

(a) 2.07 μeV (b) 2.07 meV (c) 1.035 μeV (d) 1.035 meV

43. The energies of the 4*s* and 4*p* states of potassium are –4.339 eV and –2.73 eV respectively. The ratio of $Z_{eff}$ of 4*s* and 4*p* is

(a) 0.89 (b) 1.26 (c) 1.78 (d) 2.18

44. In helium atom the total wave function of the ortho state is a

(a) singlet with spin anti-symmetric wave functions
(b) singlet with spin symmetric wave functions
(c) triplet with spin anti-symmetric wave functions
(d) triplet with spin symmetric wave functions

45. The $^{7}$Be nucleus undergoes decay to $^{7}$Li. Which Beta decay process is possible in this decay ? (Given M($^{7}$Be) = 7.016929 amu and M($^{7}$Li) = 7.016004 amu).

(a) Beta minus decay only (b) Beta plus decay only
(c) Both Beta plus and Electron capture (d) Electron capture only

46. An unstable nucleus $^{44}_{20}$Ca undergoes decay by positron emission. The daughter product of the decay is

(a) $^{43}_{20}$Ca (b) $^{43}_{19}$K (c) $^{44}_{19}$K (d) $^{45}_{20}$Ca

47. The metric tensor $g_{ik}$, defined by $ds^2 = g_{ij}\, dx^i\, dx^j$. If $ds^2 = dr^2 + r^2(d\theta)^2 + r^2 \sin^2\theta (d\phi)^2$, and

$$\Gamma_{ij,k} = \frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^k}\right],$$ then $\Gamma_{22,1}$ is

(a) $-r$ (b) $r$ (c) $r^2$ (d) $2r$

48. The solution of $\frac{\partial u}{\partial t} = 4\frac{\partial u}{\partial x}$ is

(a) $A\, e^{kx+t}$ (b) $A\, e^{k\left(\frac{x}{2}+t\right)}$ (c) $A\, e^{(x+t)}$ (d) $A\, e^{k\left(\frac{x}{4}+t\right)}$

49. If $C$ is the unit circle traversed clockwise, then the integral $\int_C \frac{dz}{z^2 - 3z}$ is

(a) 0 (b) $\frac{\pi}{3}i$ (c) $\frac{2\pi}{3}i$ (d) $-\frac{2\pi}{3}i$

50. The Fourier series of a periodic function (with period $2\pi$) is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

If $f(x)$ is defined by

$$f(x) = \begin{cases} x & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

then $a_0$ is equal to

(a) 0 (b) $2\pi$ (c) 1 (d) $-2\pi$

# SOLVED PAPER : DU 2017

PHYSICS-PH

## OBJECTIVE QUESTIONS

1. $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ and $\vec{r} \cdot \vec{r} = r^2$

We know that, $\vec{\nabla}.\left(u\vec{V}\right) = \left(\vec{\nabla}u\right).\vec{V} + u\left(\vec{\nabla}.\vec{V}\right)$

$$\vec{\nabla}\cdot\left(r^2\vec{r}\right) = \left(\vec{\nabla}r^2\right).\vec{r} + r^2\left(\vec{\nabla}\cdot\vec{r}\right) = \left(2r\,\hat{r}\right).\vec{r} + r^2(3) = 2r^2 + 3r^2 = 5r^2$$

**Correct option is (b)**

2. $\dfrac{dy}{dx} + 2xy = x$

$$\text{I.F.} = e^{\int 2x\,dx} = e^{x^2}$$

The solution to the above differential equation is

$$\therefore\ y\cdot e^{x^2} = \int x e^{x^2}\,dx = \frac{1}{2}\int e^t\,dt \qquad \left(x^2 = t \Rightarrow 2x\,dx = dt\right)$$

$$= \frac{1}{2}e^{x^2} + C$$

$$\text{or}\ \ y = \frac{1}{2} + Ce^{-x^2}$$

Using the condition $y(x=0) = 0$, we get

$$0 = \frac{1}{2} + C \Rightarrow C = -\frac{1}{2} \Rightarrow y(x) = \frac{1}{2}\left(1 - e^{-x^2}\right)$$

**Correct option is (c)**

3. $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \Rightarrow A^2 = 2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 2A$

$A^3 = (2A)A = 2A^2 = 2(2A) = 2^2 A$. Similarly $A^n = 2^{n-1}A$

$$\therefore\ e^A = I + A + \frac{1}{2!}A^2 + \frac{1}{3!}A^3 + \cdots$$

$$= I + A + \frac{1}{2!}(2A) + \frac{1}{3!}2^2 A + \cdots$$

$$= I + A\left[1 + \frac{1}{2!}(2) + \frac{1}{3!}2^2 + \frac{1}{4!}2^3 + \cdots\right]$$

$$= I + \frac{A}{2}\left[2 + \frac{1}{2!}2^2 + \frac{1}{3!}2^3 + \cdots\right] \Rightarrow e^{\hat{A}} = I + \frac{A}{2}\left[e^2 - 1\right]$$

**Correct option is (a)**

CAREER ENDEAVOUR

4. $\cos(z) = \cos(x+iy) = \cos x.\cos(iy) - \sin x.\sin(iy) = \cos x.\cosh y - \sin x.(i\sinh y)$

$\Rightarrow \text{Re}[\cos(z)] = \cos x \cosh y$

**Correct option is (a)**

5. The efficiency is given by

$$\eta = \frac{W}{Q_{in}} \Rightarrow Q_{in} = \frac{2.5\times10^4}{0.59} = 4.24\times10^4 \text{ J}$$

**Correct option is (c)**

6. The Fermi function is

$$f(E) = \frac{1}{\exp\left(\frac{E-E_F}{k_BT}\right)+1} = \frac{1}{e^2+1} \quad (\text{Given: } E-E_F = 2k_BT)$$

$\Rightarrow f(E) = 0.119 \simeq 12\%$

Therefore, the probability that an allowed states is occupied if it lies above the fermi level by $2k_BT$

$f(E) = 12\%$

**Correct option is (b)**

7. At constant pressure, we have

$$\frac{V}{T} = \text{constant}$$

$$\Rightarrow T_f = T_i\left(\frac{V_f}{V_i}\right) = T_0\left(\frac{2V_0}{V_0}\right) = 2T_0$$

$$\therefore \Delta Q = nC_P\,\Delta T = \frac{5R}{2}(2T_0 - T_0) = \frac{5R}{2}T_0$$

**Correct option is (d)**

8. Option (b) is not correct as the correct Maxwell equation is

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

**Correct option is (b)**

9. The amount of heat intake is

$$\Delta Q = \frac{m}{2}C(4T_1 - T) - \frac{m}{2}C(T - T_1)$$

$$= \frac{mC}{2}(5T_1 - 2T)$$

As the fluid is in steady state, we have

$$\frac{mv^2}{2} = \frac{mC}{2}(5T_1 - 2T)$$

$$\Rightarrow v^2 = C(5T_1 - 2T) \quad \ldots(1)$$

The entropy change in the process is

$$\Delta S = \frac{mC}{2}\ln\left(\frac{T}{T_1}\right) + \frac{mC}{2}\ln\left(\frac{T}{4T_1}\right)$$

$$= \frac{mC}{2}\ln\left(\frac{T^2}{4T_1^2}\right) = mC\ln\left(\frac{T}{2T_1}\right)$$

Since the process is irreversible, we have

$\Delta S > 0$

$\Rightarrow T > 2T_1$

Using this in equation (1), we have

$v^2 < CT_1$

$\therefore\ v_{max} = \sqrt{CT_1}$

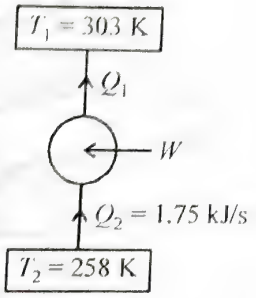
**Correct option is (d)**

10. For Carnot engine, we have

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow Q_1 = \frac{303}{258} \times 1.75 = 2.06 \text{ kJ/s}$$

$\therefore\ W = Q_1 - Q_2 = 2.06 - 1.75 = 0.31$ kJ/s




**Correct option is (b)**

11. To return at $x = 0$ after four steps, the walker takes 2 steps to right and 2 steps to left.

Therefore, probability of taking $n_1$ steps to right with probability $p$ and $n_2$ steps to left with probability $q$ is

Prob. $= {}^{n_1+n_2}C_{n_1}\, p^{n_1} q^{n_2}$

$$= {}^4C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = \frac{4!}{2! \times 2!} \times \frac{2^2}{3^4}$$

$$= \left(\frac{2}{3}\right)^3$$

**Correct option is (*)**

12. The possible configuration are :

| $-\varepsilon$ | 0 | $\varepsilon$ |
|---|---|---|
| • | • | • |
| | ••• | |

Therefore, $S = k \ln \Omega = k \ln 2$

**Correct option is (d)**

13. The area of the coil is

$= 0.05 \times 0.08$

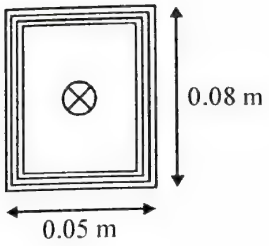
$= 40 \times 10^{-4}\ \text{m}^2$

$= 0.40 \times 10^{-2}\ \text{m}^2$

Given : Initial magnetic field $B_1 = 0.14T$

After 0.20 second magnetic field become $B_2 T$

Therefore, induced e.m.f.




$$= \frac{(B_2 - B_1) \times AN}{\Delta t} = \frac{(B_2 - 0.14) \times 0.40 \times 10^{-2} \times 420}{0.20}$$

$$\Rightarrow 1.8 = (B_2 - 0.14) \times 2 \times 10^{-2} \times 420$$

$$\Rightarrow (B_2 - 0.14) = \frac{0.9 \times 10^2}{420} = 0.21\ \text{T}$$

$$\Rightarrow B_2 = 0.35\ \text{T}$$

**Correct option is (b)**

14. In free space direction of propagation and direction of Poynting are same.

Now, we have $\vec{B} = B_1(\hat{x} - \hat{z})$

$$\Rightarrow \hat{B} = \frac{1}{\sqrt{2}}(\hat{x} - \hat{z}) = \hat{H} \text{ (In free space)}$$

And $\vec{E} = E_1(-\hat{x} + 2\hat{y} - \hat{z})$

$$\Rightarrow \hat{E} = \frac{(-\hat{x} + 2\hat{y} - \hat{z})}{\sqrt{6}}$$

Therefore, the direction of propagation

$$\hat{S} = \hat{E} \times \hat{H} = \left(-\frac{\hat{x} + 2\hat{y} - \hat{z}}{\sqrt{6}}\right) \times \frac{(\hat{x} - \hat{z})}{\sqrt{2}}$$

$$= -\frac{\hat{y} - 2\hat{z} - 2\hat{x} - \hat{y}}{\sqrt{12}} = -\frac{\hat{x} - \hat{y} - \hat{z}}{\sqrt{3}}$$

**Correct option is (a)**

15. Initially particle was at rest at the origin.

Now, we have $\vec{E} = E_0\hat{x}$ and $\hat{B} = B_0\hat{x}$

The force act on the charge particle is

$$\vec{F} = q\vec{E} = qE_0\hat{x} \Rightarrow a_x = \left(\frac{qE_0}{m}\right)$$

So particle moves along $\hat{x}$ axis with an acceleration $(qE_0/m)$.

**Correct option is (b)**

16. Given charge density $\rho = A\sqrt{r}$

According to Gauss law, $\oint \vec{E} \cdot d\vec{s} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$

For $r < a$, inside the sphere

$$E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \int_0^r \rho(r) 4\pi r^2 \, dr \Rightarrow E = \frac{1}{\varepsilon_0} \frac{2A 4\pi r^{7/2}}{7 \cdot 4\pi r^2} \Rightarrow E \propto r^{3/2}$$

**Correct option is (a)**

17. Given current density $\vec{J} = J_0 r \, \hat{z}$

According to Ampere's law, $\oint B \cdot dl = \mu_0 \iint \vec{J} \cdot d\vec{s}$

$$\Rightarrow B 2\pi d = \mu_0 J_0 \int_0^{2\pi} d\phi \int_0^d r^2 dr \qquad \text{(for inside the conductor)}$$

$$\Rightarrow \vec{B} = \frac{1}{3} \mu_0 J_0 d^2 \hat{\phi}$$

**Correct option is (c)**

18. The impedance offered by capacitor is $z = \frac{1}{j\omega c}$

$$\Rightarrow |z| = \frac{1}{\omega c} \Rightarrow |z| \propto \frac{1}{\omega}$$

So as frequency of the AC source increase, the impedance will decrease.

**Correct option is (a)**

19. Displacement current inside the capacitor during the discharge of capacitor will flow from negative to positive plate.

**Correct option is (d)**

20. Total number of possible outcomes is $6 \times 6 = 36$

(a) Probability that $S = 1$ is zero (since, $S \geq 2$)

(b) Favourable outcomes for getting a sum of 6 are. (1. 5), (2, 4), (3, 3), (4, 2), (5, 1)

Therefore, $P(S = 6) = \frac{5}{36}$

(c) Favourable outcomes for getting a sum of 2 are, (1, 1)

Therefoe, $P(S = 2) = \frac{1}{36}$

(d) Favourable outcomes for getting a sum of 3 are, (1, 2), (2, 1)

Therefore, $P(S = 3) = \frac{2}{36} = \frac{1}{18} = 2$

**Correct option is (b)**

21. The given normalized wavefunction is $|\psi\rangle = \frac{3|\psi_1\rangle + 4|\psi_2\rangle}{5}$

$$\langle \hat{A} \rangle_{|\psi\rangle} = \langle \psi | \hat{A} | \psi \rangle$$

$$= \left( \frac{3}{5}\langle \psi_1 | + \frac{4}{5}\langle \psi_2 | \right) \hat{A} \left( \frac{3}{5}|\psi_1\rangle + \frac{4}{5}|\psi_2\rangle \right)$$

$$= \frac{9}{25}\langle \psi_1 | \hat{A} | \psi_1 \rangle + \frac{12}{25}\left( \langle \psi_1 | \hat{A} | \psi_2 \rangle + \langle \psi_2 | \hat{A} | \psi_1 \rangle \right) + \frac{16}{25}\langle \psi_2 | \hat{A} | \psi_2 \rangle$$

Now, $\hat{A}|\psi_1\rangle = |\psi_2\rangle$ ; $\hat{A}|\psi_2\rangle = |\psi_1\rangle$

$$= \frac{9}{25}\langle \psi_1 | \psi_2 \rangle + \frac{12}{25}\left[ \langle \psi_1 | \psi_1 \rangle + \langle \psi_2 | \psi_2 \rangle \right] + \frac{16}{25}\langle \psi_2 | \psi_1 \rangle$$

$$= \frac{12}{25} \times 2 = \frac{24}{25} = 0.96$$

**Correct option is (d)**

22. When a particle of charge $q$ is accelerated through a potential difference $V$, the change in potential energy will be $qV$.

Here, kinetic energy, $\left( \frac{p^2}{2m} \right) = qV \Rightarrow$ momentum $(p) = \sqrt{2mqV}$

Now, de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$

**Correct option is (a)**

23. For a wavefunction of a hydrogen atom state having azimuthal quantum number $m$, $\psi(\vec{r}) \propto e^{im\phi}$

Here, $\psi(\vec{r})$ is independent of $\phi$ $\therefore$ $m = 0$

Now, $\hat{L}_z |\psi_{n,l,m}\rangle = m\hbar |\psi_{n,l,m}\rangle$

$\therefore$ The value $\hat{L}_z$ for the given $\psi$ will be zero.

**Correct option is (d)**

24. We can say from position-momentum uncertainty relation that, $\Delta r \Delta p \sim \hbar$

$\Rightarrow \Delta p \sim \frac{\hbar}{\Delta r}$ ; $p \sim \Delta p$ for low energy states

Therefore, kinetic energy $= \frac{p^2}{2m} \simeq \frac{\hbar^2}{2m\Delta r^2}$

$$\simeq \frac{6.626^2 \times 10^{-68}}{4\pi^2 \times 2 \times 9.1 \times 10^{-31} \times 10^{-30} \times 1.6 \times 10^{-19}} \text{ eV} \simeq 0.3 \times 10^{-13} \text{ eV}$$

Therefore, the average kinetic energy is of the order of $0.511 \times 10^{-13}$ eV

**Correct option is (c)**

25. The close loop gain is given by $A_{CL} = \dfrac{A}{1+A\beta}$

Therefore, $\dfrac{\partial A_{CL}}{A_{CL}} = \dfrac{\partial A/A}{1+A\beta} = \dfrac{10\,\%}{1+400\times 0.05}$, where $\beta$: feedback factor and A : open-loop-gain

$$\Rightarrow \frac{\partial A_{CL}}{A_{CL}} \cong 0.5\,\%$$

**Correct option is (c)**

26. The LSB of n-bit digital to analog converter is given by

$$S = \text{Resolution} = \frac{V_H - V_L}{2^n - 1} \Rightarrow S = \frac{12.8-0}{2^8-1} \quad (n = 8 \text{ bit})$$

$S \cong 0.050$ volt

**Correct option is (a)**

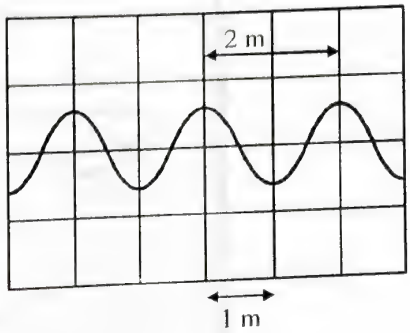
27. Given: value of each division = 1 m sec.
Therefore, the time period of signal,

$T = 2 \times 1$ milli sec

$= 2$ milli sec

Therefore, the frequency of the signal,

$$f = \frac{1}{T} = \frac{1}{2 \text{ m sec}} = 500 \text{ Hz}$$




**Correct option is (d)**

28. $[A\ \ 3\ \ B]_{16} \longrightarrow$ Base-16 (Hexadecimal)

$\downarrow\ \ \downarrow\ \ \downarrow$

$10\ \ 3\ \ 11$

$10\times 16^2 + 3\times 16^1 + 11\times 16^0 = (2619)_{10}$

**Correct option is (c)**

29. $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$Y = \bar{A}\bar{C}[\bar{B} + B] + A\bar{C}[\bar{B} + B]$

$Y = \bar{A}\bar{C} + A\bar{C}$

$Y = \bar{C}[\bar{A} + A]$ $\quad \boxed{x + \bar{x} = 1}$ Axiom

$Y = \bar{C}$

**Correct option is (d)**

30. Let the spaceship be $S'$ frame and the Earth be $S$ frame.

Length of spaceship in its rest frame, $L_0 = 1150$ m.

Length of spaceship as observed from Earth, $L = 325$ m.

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow 325 = 1150\sqrt{1-\frac{v^2}{c^2}} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \left(\frac{325}{1150}\right) = \left(\frac{13}{46}\right)$$

Total energy, $E = mc^2 = \dfrac{m_0c^2}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{m_0c^2}{13/46} = \dfrac{46}{13}m_0c^2$

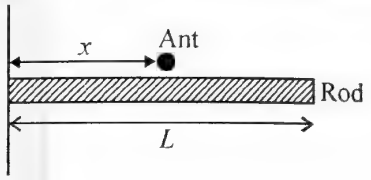
$$E = \frac{46}{13} \times 75 \times 9 \times 10^{16} \simeq 2.4 \times 10^{19} \text{ J}$$

**Correct option is (c)**

31. Moment of inertia of (ant + rod) at any time $t$,

$$I = \frac{1}{3}ML^2 + mx^2$$

As $x$ increases, the moment of inertia $I$ also increases. Due to conservation of angular momentum, angular velocity decreases.




Kinetic energy, K.E $= \dfrac{1}{2}I\omega^2 = \dfrac{L^2}{2I}$ ($L$ is conserved)

$\therefore$ K.E $\propto \dfrac{1}{I}$

Since, moment of inertia increases, K.E. decreases.

**Correct option is (a)**

32. Gravitational field at the surface, $g = \dfrac{GM}{R^2}$

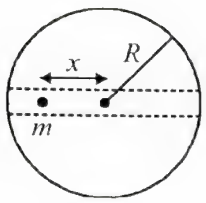
Let $\rho$ be the density of earth, $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$

For a uniformly distributed spherical mass, the gravitational field at a distance $x$ from the centre

$E_g = -\dfrac{4\pi Gx}{3}\rho$ (negative sign is due to attractive field)

Force on the ball of mass $m$,

$$F = mE_g = -\frac{4\pi G\rho}{3}mx = -\frac{4\pi}{3}\left(\frac{gR^2}{M}\right)\frac{M}{\left(\frac{4}{3}\pi R^3\right)}mx$$




$$= -\frac{gm}{R}x \equiv -kx \quad \text{(A case of SHM)}$$

$\therefore \quad k = \dfrac{gm}{R}$

Angular frequency, $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{R}}$

Time period of oscillation, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R}{g}}$

Time taken by ball to reach the centre of earth, $t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{R}{g}}$

**Correct option is (c)**

33. Acceleration of the block when going upwards,

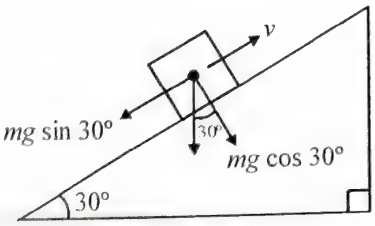
$a = -g(\sin 30° + \mu \cos 30°)$

$= -10\left(\frac{1}{2} + \frac{\sqrt{3}}{6} \times \frac{\sqrt{3}}{2}\right)$

$= -7.5 \text{ m/s}^2$

Distance covered before stopping,

$S = \frac{u^2}{2a} = \frac{9}{2 \times 7.5} = 0.6 \text{ m}$




Time taken to stop, $t = \frac{u}{a} = \frac{3}{75} = 0.4 \text{ s}$

Given total time, $T = 1.5 \text{ s}$

Remaining time, $t' = T - t = (1.5 - 0.4)\text{s} = 1.1 \text{s}$

Now, acceleration of block when going downwards,

$a' = g(\sin\theta - \mu\cos\theta)$

$= 10\left(\frac{1}{2} - \frac{\sqrt{3}}{6}\frac{\sqrt{3}}{2}\right) = 2.5 \text{ m/sec}^2$

Distance covered in time, $t'$, $S' = \frac{1}{2}a't'^2 = \frac{1}{2} \times 2.5 \times 1.1^2 = 1.5125 \text{ m}$

Net displacement, $\Delta S = +S - S' = -0.9 \text{ m}$ from original position

**Correct option is (a)**

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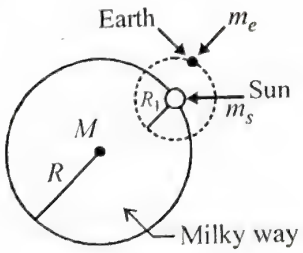
34. Time period of sun around Milky way, $T = \frac{2\pi R}{v}$




Equation of motion of Sun, $\frac{m_s v^2}{R} = \frac{G m_s M}{R^2}$

$\Rightarrow v = \sqrt{\frac{GM}{R}}$

$T = 2\pi R\sqrt{\frac{R}{GM}} = 2\pi\sqrt{\frac{R^3}{GM}}$ ... (1)

Similarly for time period of earth's orbit around Sun,

$T_1 = \frac{2\pi R_1}{v_1} = 2\pi\sqrt{\frac{R_1^3}{Gm_s}}$ ... (2)

Dividing Eq. (1) by Eq. (2), we get

$\frac{T}{T_1} = \sqrt{\frac{R^3 m_s}{R_1^3 M}} \Rightarrow \frac{M}{m_s} = \left(\frac{R}{R_1}\right)^3 \left(\frac{T_1}{T}\right)^2$

Putting the values of $R = 25000$ light years

$= (25000 \times 365 \times 24 \times 60)$ light minutes

$= 1.3 \times 10^{10}$ light minutes

$R_1 = 8$ light minutes, $T_1 = 1$ year

$T = 170$ million years $= 17 \times 10^7$ years

$$\frac{M}{m_s} = \left(\frac{1.3 \times 10^{10}}{8}\right)^3 \left(\frac{1}{17 \times 10^7}\right)^2 \simeq 1.5 \times 10^{11}$$

**Correct option is (a)**

35. Reduced mass $\mu = \frac{m^2}{2m} = \frac{m}{2}$

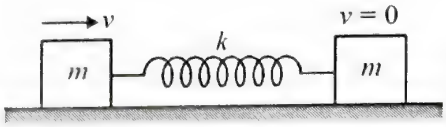
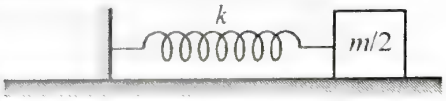


Fig. (1)

$$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k}{m/2}} = \sqrt{\frac{2k}{m}}$$



Equivalent system

Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

The time when the second block will come to rest is when the original configuration is repeated as shown in figure (1),

$$t = T = 2\pi\sqrt{\frac{m}{2k}} = \pi\sqrt{\frac{2m}{k}}$$

**Correct option is (a)**

36. Here, Number of slits are N = 3.

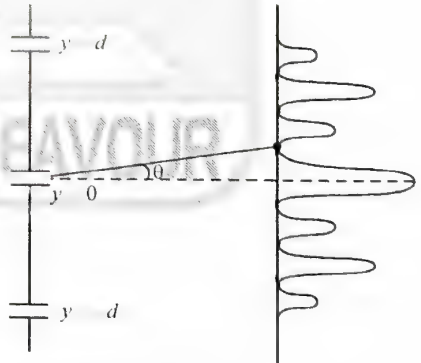
Position of secondary minima of diffraction pattern

$$d\sin\theta = \frac{P\lambda}{N} = \frac{P\lambda}{3}$$

Position of first minima, P = 1

$$\sin\theta = \frac{\lambda}{3d} \Rightarrow \theta = \sin^{-1}\frac{\lambda}{3d}$$

**Correct option is (a)**




37. Let wave produced by fork is $y_1 = A\sin 2\pi\nu_1 t$ and $y_2 = A\sin 2\pi\nu_2 t$

Superposition of waves $y = y_1 + y_2$

When two waves of different frequencies superpose, intensity vary with frequency $\nu_1 - \nu_2$ and particle oscillates with $\frac{\nu_1 + \nu_2}{2}$.

Given : $\nu_1 - \nu_2 = 1$ Hz

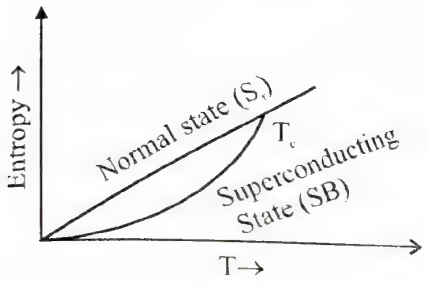
$$\frac{\nu_1 + \nu_2}{2} = 512 \text{ Hz} \Rightarrow \nu_1 = 512.5 \text{ Hz and } \nu_2 = 511.5 \text{ Hz}$$

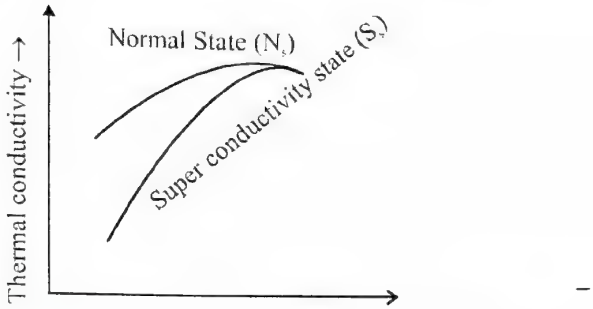
**Correct option is (d)**

38. In all superconductors, the entropy decreases marketly on cooling below the temperature ($T_c$). Entropy is a measure of disorder of the system and hence the observed decrease in entropy between normal state and superconducting state tells that "superconducting state is more ordered than the normal state."

**Thermal conductivity:**

The thermal conductivity of superconductors undergoes a continuous change between the two phases and is usually lower in superconductivity phase.




According to the above discussion, we can say that during normal to superconducting state transition, both entropy and thermal conductivity decreases.

**Correct option is (b)**

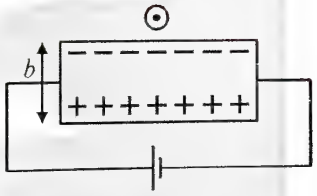
39. Suppose silicon water has thickness $d$ and width $b$.

The Hall voltage is given by

$$V_H = vBd = \frac{IB}{nebd} = \frac{IB}{ned}$$

$$= \frac{1\times10^{-3}\times0.2}{10^{23}\times1.6\times10^{-19}\times200\times10^{-6}}$$

$$= \frac{0.2}{1.6}\times10^{-3} = 62.5\ \mu V$$




**Correct option is (c)**

40. According Kronig-Penney model.

The scattering power of the potential barrier is given by $P\cdot\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$

Since, right hand become $\pm1$ for values of $k=(n\pi/a)$

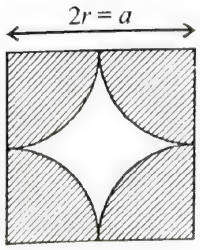
Hence, the discontinuities in the $E$ versus $k$ graph occur at $k=\frac{n\pi}{a}$, where $n=\pm1,\pm2,\pm3$.

**Correct option is (a)**

41. The number of atom in unit simple cubic cell = 8×(1/8) = 1.

The volume of the atom,

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \left(\frac{\pi a^3}{6}\right)$$




The volume of the cell is $a^3$.

Therefore, the fraction of volume occupied by atom is

$$\frac{\text{Volume occupied by the atom in unit cell}}{\text{Volume of the unit cell}} = \frac{\pi a^3/6}{a^3} = \frac{\pi}{6}$$

**Correct option is (c)**

42. In superconductor the energy gap is given by

$= 3.52\, k_B T_c = 3.52 \times 1.38 \times 10^{-23} \times 3.4\ \text{J} = 10.32 \times 10^{-4}\ \text{eV} = 1.032\ \text{meV}$

**Correct option is (d)**

43. The energy of an electron in Alkali atom is given by

$$E_{n,l} = -\frac{m z_{\text{eff}}^2 e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$$

$$\therefore \quad \frac{E_{4,1}}{E_{4,0}} = \frac{\left(z_{\text{eff}}^2\right)_{4p}}{\left(z_{\text{eff}}^2\right)_{4s}} \Rightarrow \frac{(z_{\text{eff}})_{4s}}{(z_{\text{eff}})_{4p}} = \sqrt{\frac{E_{4,0}}{E_{4,1}}} = \sqrt{\frac{2.73}{4.339}} = 0.793$$

**Correct option is (d)**

44. The He atom is a system of two electrons. So, according to Pauli's requirement that its total wave function must be antisymmetric.
In ortho helium state helium atom have parallel electron spins which give triplet state. So in ortho helium spin part are symmetric.
**Correct option is (d)**

45. The $^7$Be nucleus undergoes decay to $^7$Li.
This reaction may be followed by position emission or electron capture.
The energy yield of the reaction is given by

$$\text{Energy yield} = \left[M\left({}^7_4Be\right) - M\left({}^7_3Li\right)\right] \times 931.5\ MeV$$

$$= \left[7.016929 - 7.016004\right] \times 931.5\ MeV = 0.000925 \times 931.5\ MeV = 0.86\ MeV$$

For positron emission, this energy yield should be greater than or equal to 1.02 MeV. Hence, decay of ${}^7_4Be$ nucleus via positron emission of not possible.

For electron capture, the energy yield should be just greater than zero. Hence, decay of ${}^7_4Be$ nucleus is possible only via electron capture.
The reaction of this decay process is

$$\underset{\text{(Parent nucleus)}}{{}^7_4Be} + \underset{\text{(Electron)}}{{}^0_{-1}e} \longrightarrow \underset{\text{(Daughter nucleus)}}{{}^7_3Li} + \underset{\text{(Neutrino)}}{\nu}$$

**Correct option is (d)**

46. In positive β-decays, positron is emitted, $p \rightarrow n + \nu + e^+$

In this process, atomic number of product element is reduced by 1, therefore, ${}^{44}_{20}\text{Ca}$ is converted to ${}^{44}_{19}\text{K}$.

**Correct option is (c)**

47. $g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$

$$\Gamma_{ij,k} = \frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^k}\right]$$

$$\Gamma_{22,1} = \frac{1}{2}\left[\frac{\partial g_{21}}{\partial \theta} + \frac{\partial g_{21}}{\partial \theta} - \frac{\partial g_{22}}{\partial r}\right] = \frac{1}{2}\left(-\frac{\partial}{\partial r}(r^2)\right)$$

or $\Gamma_{22,1} = -r$

**Correct option is (a)**

48. Let $u(x,t) = Ae^{k\left(\frac{x}{4}+t\right)} \Rightarrow \frac{\partial u}{\partial t} = ku(x,t)$.

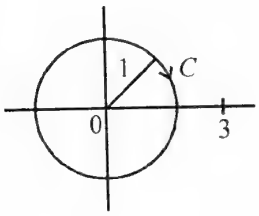
Also, $\frac{\partial u}{\partial x} = \frac{k}{4}u(x,t) \Rightarrow 4\frac{\partial u}{\partial x} = ku(x,t)$

$\Rightarrow \frac{\partial u}{\partial t} = 4\frac{\partial u}{\partial x}$

**Correct option is (d)**

49. $f(z) = \frac{1}{z^2 - 3z} = \frac{1}{z(z-3)}$ has simple poles at z = 0, 3.

Only z = 0, lies within the given contour $|z| = 1$.




Residue of $f(z)$ at $(z = 0) = \underset{z\to 0}{Lim}(z-0)\frac{1}{z(z-3)} = -\frac{1}{3}$

Therefore, $\int_C \frac{dz}{z^2 - 3z} = -2\pi i \times \left(-\frac{1}{3}\right) = \frac{2\pi i}{3}$

**Correct option is (c)**

50. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

The co-efficient $a_0$ in the Fourier series is given by

$$a_0 = \frac{2}{2\pi}\int_{-\pi/2}^{3\pi/2} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi/2}^{\pi/2} x\,dx + \int_{\pi/2}^{3\pi/2}(\pi - x)\,dx\right]$$

$$= \frac{1}{\pi}\left[\left\{\frac{x^2}{2}\right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \left\{\pi x - \frac{x^2}{2}\right\}_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right] = \frac{1}{\pi}\left[0 + \left\{\left(\frac{3\pi^2}{2} - \frac{9\pi^2}{8}\right) - \left(\frac{\pi^2}{2} - \frac{\pi^2}{8}\right)\right\}\right] = 0$$

**Correct option is (a)**

# PAPER : DU 2018
## PHYSICS-PH

**INSTRUCTIONS:**

(*i*) This test booklet contains 50 Objective type questions.

(*ii*) Each question carries **4 marks. 1 mark** will be deducted for each incorrect answer. Only one box should be marked and the marking of more than one box will be treated as wrong answer.

## OBJECTIVE QUESTIONS

1. A particle is confined in a one dimensional potential box with impenetrable walls at $x = \pm a$. Its energy eigenvalue is 2 eV and corresponds to the eigenfunction of the first excited state. The lowest possible energy of the particle is
(a) 2.0 eV (b) 0.5 eV (c) 4.0 eV (d) 1.0 eV

2. The net magnetic moment of Iron atom in BCC crystal (a = 2.857 $A^0$) is 2.2 $\mu_s (\mu_s = 9.273\times10^{-24}\, Am^2)$. The saturation magnetization of Fe at 0 K is
(a) 1750 $Am^{-1}$ (b) 1750 $Am^{-2}$ (c) 0 (d) 1749.6 $kAm^{-1}$

3. Three dice are thrown simultaneously. The probability that the same number will appear on all the three dice is
(a) 1/36 (b) 1/6 (c) 1/18 (d) 1/108

4. Let us model a star as a spherical black body at temperature T which radiates to distant points which are at absolute zero. Let us further model a dust cloud around the star as a black sphercial shell whose temperature $T_s$ is determined by radiative equilibrium. What is $T_s$?
(a) $2^{-1/2}T$ (b) $2^{-1/4}T$ (c) T (d) T/2

5. Consider a Carnot reversible heat engine working between the temperatures of melting ice and steam. The efficiency of this heat engine will be approximately equal to
(a) 37% (b) 27% (c) 100% (d) None of the above

6. Which one of the following is not a characteristic of CMOS configuration?
(a) CMOS devices have higher noise-margins.
(b) CMOS devices have low input impedances.
(c) CMOS devices have much lower transconductance than bipolar devices
(d) CMOS devices dissipate lower static power than bipolar devices

7. the value of the integral

$$\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)}$$

(a) $2\pi/3$ (b) $\pi/2$ (c) $\pi/4$ (d) $3\pi/4$

8. Given $z = x + iy$, the contour integration $\oint_C \frac{dz}{z}$ is equal to (where $C$ is any anti-clockwise contour going around the origin)
(a) $-\pi i$ (b) $2\pi i$ (c) $\pi i$ (d) 0

9. Solution of the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 8e^x$ is

(a) $Axe^{3x} + Be^x$ (b) $(Ax + B)e^{3x} + 2e^x$

(c) $(Ax^2 + Bx)e^{4x} + 2e^{2x}$ (d) $(Ax + B)e^{3x} + 8e^x$

10. Train A and B are running on adjacent rail tracks in the same direction with speeds 43 m/s and 10 m/s, respectively, with train B ahead of train A. When train A is 1000 m behind train B, train A sounds a whistle with frequency 270 Hz in its rest frame. Assuming still air and a speed of sound of 330 m/s, the frequency of the sound as meansured by a passenger at rest in train B is equal to

(a) 301 Hz (b) 280 Hz (c) 310 Hz (d) 290 Hz

11. An Earth-like exoplanet orbits around a star having a mass 1/27 times the mass of Sun. This exoplanet completes one revolution around the star in 365 days just like Earth. Assuming that orbits of both Earth as well as this exoplanet are circular, the distance of the exoplanet from star is

(a) one-ninth of the Earth-sūn distance (b) 3 times the Earth-Sun distance

(c) same as the Earth-Sun distance (d) One-third of the Earth-Sun distance

12. Consider eight electrons in a one dimensional box of length '$a$' extending from $x = 0$ to $x = a$. What is the minimum allowed total energy using Pauli's exclusion principle for the system ($m$ = mass of electron)

(a) $\frac{8h^2}{ma^2}$ (b) $\frac{15h^2}{2ma^2}$ (c) $\frac{15h^2}{4ma^2}$ (d) $\frac{10h^2}{ma^2}$

13. For an electromagnetic wave travelling in free space, given by $\vec{E} = E_m \sin(\omega t - \beta z)\hat{y}$, the magnetic field $\vec{B}$ will be given by

(a) None of the above (b) $\vec{B} = -\frac{E_m\beta}{\omega}\sin(\omega t - \beta z)\hat{x}$

(c) $\vec{B} = -\frac{E_m\beta}{\omega}\cos(\omega t - \beta z)\hat{z}$ (d) $\vec{B} = \frac{E_m\beta}{\omega}\sin(\omega t - \beta z)\hat{y}$

14. The magnetic field required to bend a non-relativistic charged particle moving with velocity $v$ into an arc of radius of curvatrue $R$ is

(a) inversely proportional to $v^2$ and directly proportional to $R^2$

(b) inversely proportional to $v$ and directly proportional to $R$

(c) directly proportional to $v$ and inversely proportional to $R$

(d) directly proportional to $v^2$ and inversely proportional to $R^2$

15. A atom is placed in a magnetic field of suffcient strength for splitting the 3p level. The number of levels resulting due to splitting will be

(a) 1 (b) 4 (c) 3 (d) 2

16. In 3-dimensional space, a particle of mass $m$ moves in a potential $B \sin 2\alpha r$ where $r$ is the distance of the particle from the origin, B and $\alpha$ are real constants. Which of the following statement is correct?

(a) The motion is periodic in $r$ with an oscillation length scale $2\pi/\alpha$.

(b) The trajectory of the particle is always confined to a plane passing through the origin.

(c) The motion is periodic in $r$ with an oscillation length scale $\pi/\alpha$.

(d) Symmetry implies that a radial momentum $p_r$ is conserved because of the periodic nature of the potential.

17. Fourier transform of any fuention $f(x)$ is defined to be

$$F(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\exp(-ikx)dx$$

Then, given a function $g(x) = 1$ for $|x|<1$ and zero otherwise, its Fourier transform G(k) is

(a) $\sqrt{\frac{2}{\pi}}\frac{\sin(k)}{k}$ (b) $\sqrt{\frac{1}{\pi}}\frac{\exp(k)}{k}$ (c) $\sqrt{\frac{2}{\pi}}\cos(k)$ (d) $\sqrt{\frac{1}{\pi}}\frac{\exp(-k)}{k}$

18. The fine structure splitting of the 2p state of carbon and oxygen atoms in the L-S coupling scheme will have the energy respectively as

(a) ${}^3P_2 > {}^3P_1 > {}^3P_0$ and ${}^3P_2 > {}^3P_1 > {}^3P_0$ (b) ${}^3P_2 < {}^3P_1 < {}^3P_0$ and ${}^3P_2 < {}^3P_1 < {}^3P_0$

(c) ${}^3P_2 > {}^3P_1 > {}^3P_0$ and ${}^3P_2 < {}^3P_1 < {}^3P_0$ (d) ${}^3P_2 < {}^3P_1 < {}^3P_0$ and ${}^3P_2 > {}^3P_1 > {}^3P_0$

19. The Boolean expression $(A+B)(A+\bar{B})(\bar{A}+B)$ is equivalent to

(a) $A\bar{B}$ (b) $\bar{A}B$ (c) $\overline{AB}$ (d) $AB$

20. Consider a rotating spherical planet such that the effective gravitational attraction at the equator is only 75% of that at the pole. If the linear velocity of a point on the equator is $v_0$, what is the escape velocity for a polar particle?

(a) $v_0\sqrt{8}$ (b) $v_0\sqrt{2}$ (c) $2v_0$ (d) $4v_0$

21. If $f(x)$ is a non-negative continuous function for all x, such that $f(x)+f(x+1/2)=1, 0\le x\le\frac{1}{2}$, then $\int_0^1 f(x)dx$ is equal to

(a) 1 (b) 1/4 (c) 1/2 (d) 2

22. A charged particle of charge $q$ and mass $m$ enters with initial velocity $u$ along the $x$ direction into a region defined by $0\le x\le L$ in which a uniform electrci field $\vec{E}$ is applied along the y direction. The charged particle gets deflected by distance $D$ along the $y$ direction when it emerges out of the region. Which of the following statements is false.

(a) $D$ is directly proportional to $u$ (b) $D$ is directly proportional to $q/m$

(c) $D$ is directly proportional to $L^2$ (d) $D$ is directly proportional to $\vec{E}$

Which of the following current densities can generate the magnetic vector potential $\vec{A}=y^2\hat{x}+x^2\hat{y}$?

(a) $-\frac{2}{\mu_0}(\hat{x}+\hat{y})$ (b) $-\frac{2}{\mu_0}(\hat{x}-\hat{y})$ (c) $\frac{2}{\mu_0}(x\hat{x}+y\hat{y})$ (d) $\frac{2}{\mu_0}(x\hat{x}-y\hat{y})$

Let $z=x+iy$ where $x$ and $y$ are real and $i=\sqrt{-1}$. The points $(x, y)$ in the plane for which $\frac{z+i}{z-i}$ is purely imaginary lie on

(a) a hyperbola (b) an ellipse (c) a circle (d) a straight line

25. Thermal runaway in a transistor biassed in the active region is due to
(a) change in reverse collector saturation current due to rise in temperature
(b) breakdown under reverse biasing
(c) changes in β in which increases with temperature
(d) base-emitter voltage $V_{BE}$ which decreases with rise in temperatrue

26. Life-time of an excited state of an atom that always jumps to the ground state by emitting a photon spontaneously is $10^{-9}$ second. The frequency of the emitted photon is uncertain at least by an amount.
(a) $10^4$ Hz. (b) $10^8$ Hz. (c) $10^5$ Hz. (d) $10^6$ Hz.

27. In a hydrogen-like atom, an electron is bound to a heavy nucleus containing $Z$ number of protons. In the framework of Bohr's atomic model, $mv_n r_n = n\hbar$, 1, 2, 3, ....., where $m$, $v_n$ and $r_n$ are the electron's mass, speed and radius, respectively, corresponding to the n- th orbit. If $a_0 = \frac{\hbar^2}{me^2}$ then.

(a) $v_n = \frac{ze^2}{n\hbar}$ and $r_n = \frac{n^2}{z}a_0$ (b) $v_n = \frac{ze^2}{n\hbar}$ and $r_n = na_0$

(c) $v_n = \frac{zne^2}{\hbar}$ and $r_n = \frac{n}{z}a_0$ (d) $v_n = \frac{e^2}{n\hbar}$ and $r_n = \frac{n}{z}a_0$

28. Four moles of an ideal gas undergo a reversible isothermal compression at 20 °C when 1850 J of work is done on the gas. The change in entropy of the gas in this process is
(a) −1.6 J/K (b) +1.6 J/K (c) + 6.3 J/K (d) −6.3 J/K

29. The degeneracy of the lowest Landau level
(a) Is constant with the magnetic field (b) Varies as function of temperatrue
(c) Varies linearly with the magnetic field (d) Varies exponentically with the magnetic field

30. The fraction of electrons excited across the energy gap of Silicon ($E_g$ = 1.1 eV) at room temperature (300 K)is
(a) $4 \times 10^{-12}$ (b) $7 \times 10^{-18}$ (c) $1.7 \times 10^{-12}$ (d) $5.712 \times 10^{-10}$

31. A large and spherical soap film of thickness $d$ has a refractive index 4/3. A narrow beam of yellow light ($\lambda = 6400$Å) is incident on the film at an angle of 30°. What is the value of d for which a constructive second order interference would occur for the reflected wave?
(a) 4800 Å (b) 3900 Å (c) 5200 Å (d) 5500 Å

32. The real part of an analytic complex function is $u(x, y) = x^2 - y^2$. The imaginary part of the function is then
(a) $2xy$ (b) $x^2 - y^2 - 2xy$ (c) $x^2 + y^2$ (d) $x^2 + y^2 + xy$

33. A uniformly charged ring of total charge Q and of radius R, rotate about an axis perpendicular to its plane and passing through its centre, with angular velocity ω. The magnetic field produced at the centre of the ring is

(a) $\frac{3\mu_0 Q\omega}{2\pi R}$ (b) $\frac{3\mu_0 Q\omega}{4\pi R}$ (c) $\frac{\mu_0 Q\omega}{4\pi R}$ (d) $\frac{\mu_0 Q\omega}{2\pi R}$

34. A child of mass 3m sits on a swing, the base of which has mass m. while the rope (of length $\ell$) has a negligible mass. An adult pulls back the swing (with the child) until the rope makes an angle of half-a-radian with the vertical, and then. pushes with a force mg along the arc of a circle until the rope is exactly vertical, and release it.If the rope always taut, for how long did the adult push?

(a) $\pi\sqrt{\frac{2\ell}{3g}}$ (b) $\frac{\pi}{3}\sqrt{\frac{\ell}{g}}$ (c) $\sqrt{\frac{\ell}{g}}$ (d) $\sqrt{\frac{2\ell}{3g}}$

The normalized wave functions $\psi_1$ and $\psi_2$ correspond to the ground state and the first excited states of a particle in a particle in a potential. The operator $\hat{A}$ acts on the wavefunctions as $\hat{A}\psi_1 = \psi_2$ and $\hat{A}\psi_2 = \psi_1$. The expection value of the operator $\hat{A}$ for the state $\psi = (3\psi_1 + 4\psi_2)/5$ is

(a) 0 (b) 0.96 (c) 0.75 (d) 0.32

A particle of mass m is confined in the ground state of a one dimensional box extending from x = –2L to x = +2L. The wave function of the particle in this state is

$$\psi(x) = \psi_0 \cos\left(\frac{\pi x}{4L}\right).$$

where $\psi_0$ is a constant. The energy eigenvalue corresponding to this state is

(a) $\frac{\hbar^2\pi^2}{32mL^2}$ (b) $\frac{\hbar^2\pi^2}{16mL^2}$ (c) $\frac{\hbar^2\pi^2}{4mL^2}$ (d) $\frac{\hbar^2\pi^2}{2mL^2}$

Let C be the unit circle. travelled counterclockwise. Evaluate

$$\oint_C \left[(e^{-x^2} - y^3)dx + x^3 dy\right]$$

(a) $\sqrt{\frac{\pi}{2}} - 2$ (b) $\sqrt{\frac{\pi}{2}} + 2$ (c) $2\pi$ (d) $\frac{3\pi}{2}$

Consider a system of 3 non-interacting identical fermions. Each particle can be in any one of four states whose energies are $-\varepsilon$, 0, $+\varepsilon$ and $+2\varepsilon$. Find the entropy of the system if its total energy E = 0.

(a) $kln2$ (b) 0 (c) $kln3$ (d) $kln4$

39. A uniform magnetic field $B$ points along the $z$ axis. At time $t = 0$, a point charge $q$ of mass $m$ is released from the origin with a velocity $\vec{v}$ making an angle 30° with the z-axis. When the point charge comes back again at $x = 0$, the $z$-coordinate of the charge is

(a) $\frac{2\pi m|v|}{\sqrt{3}qB}$ (b) $\frac{\sqrt{3}\pi m|v|}{2qB}$ (c) $\frac{\pi m|v|}{qB}$ (d) $\frac{\pi m|v|}{2qB}$

40. A thin, uniform and rigid rod $R$ of mass $M$ and length $L$ rests on a flat frictionless table top. A tiny piece of wood of mass $m = 0.01M$ sliding on the table towards $R$ with a velocity $\vec{v}$ perpendicular to the length of the rod hits one end of $R$ and gets stuck. The speed of the centre of mass and angular speed of the resulting system are respectively given by

(a) $\frac{v}{100}$ and $\frac{2v}{101L}$ (b) $\frac{v}{101}$ and $\frac{6v}{103L}$ (c) $\frac{v}{101}$ and $\frac{2v}{100L}$ (d) $\frac{v}{100}$ and $\frac{6v}{101L}$

41. The temperature dependence of the magnetic susceptibility ($\chi$) of a paramagnetic material, with Curie temperatrue ($T_c$), is given by :

(a) $C/(T+T_c)$ for $T > T_c$ (b) $C/(T-T_c)$ for $T < T_c$

(c) $C/(T+T_c)$ for all temperatures (d) $C/(T-T_c)$ for $T > T_c$

42. A semiconductor has $10^{15}$ electrons/cm$^3$ ($\mu_e = 5000$ cm$^2$/V.s) and $5\times10^{14}$ holes/cm$^3$ ($\mu_h = 400\,cm^2/V.s$). If a field of 100V/cm is applied to it, total conductivity is

(a) 83.2 $(\Omega\ cm)^{-1}$ (b) 0.832 $(\Omega\ cm)^{-1}$ (c) 0.832 $(m\Omega\ cm)^{-1}$ (d) 8.32 $(m\Omega\ cm)^{-1}$

43. For a mole of ideal gas at $T = 35°C$, what is the work done for an isothermal expansion from a volume $V_0$ to $10\ V_0$?

(a) $6 \times 10^3$ J (b) $3 \times 10^3$ J (c) $10^3$ J (d) $10^4$ J

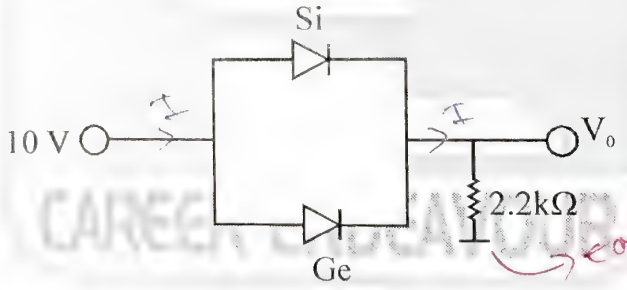
44. An ideal monoatomic gas, initially at $T = 20°C$, expands adiabatically from a volume $V_0$ to $5V_0$. Then the final temperature is

(a) –20° C (b) –33° C (c) –173° C (d) 113° C

45. Consider a body of mass 5g, which is heated from 100 K to 300 K. The specific heat for the body is 0.1 cal/g/degree. The total change in the entropy of the body is approximately equal to

(a) 0.5 cal/degree (b) 100 cal/degree (c) 50 cal/degree (d) 0 cal/degree

46. Two diodes (Si and Ge) are connected in parallel as shown in the figure. Determine the output voltage $V_0$. The cut-in voltage for Si and Ge diodes are 0.7 V and 0.2 V, respectively.




(a) –9.3 V (b) –9.8 V (c) + 9.3 V (d) +9.8 V

47. Let $f : R \to R$ be given by

$$f(x) = |x^2 - 1|, x \in R$$

Then

(a) $f$ is discontinuous a $x = \pm 1$

(b) $f$ has local maxima at $x = 0$ but no local minimum

(c) $f$ has local minima a $x = \pm 1$ and a local maxima at $x = 0$

(d) $f$ has local minima at $x = \pm 1$ but no local maximum

48. An infinite wire having uniform charge per unit length $\lambda$ (where $\lambda$ is negative) lies parallel to an infinite grounded conducting plane. The perpendicular distance between the wire and the plane is d. The maximum value of charge density induced on the plane is

(a) $\frac{|\lambda|}{\pi d}$ (b) $\frac{|\lambda|}{2\pi d}$ (c) $\frac{|2\lambda|}{\pi d}$ (d) $\frac{|\lambda|}{4\pi d}$

Two tunning forks with natural frequencies $\nu_1$ and $\nu_2$ respectively, are struck at the same time with equal force. The intensity, of the resulting sound, waxes and wanes with time period of 1.5 seconds while the frequency of the sound is 256 Hz. Hence, $\nu_1$ and $\nu_2$ . respectively, are

(a) 256.333 Hz and 255.667 Hz
(b) 256.55 Hz and 255.55 Hz
(c) 256 Hz and 256.5 Hz
(d) 256.5 Hz and 257 Hz

A blackbody at a temperature of 6000 K emits radiation whose intensity spectrum peaks at 600 nm. If the temperatrue is reduced to 300 K, the spectrum will peak at

(a) 12 nm
(b) 120 μm
(c) 120 nm
(d) 12 μm

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# SOLVED PAPER : DU 2018

PHYSICS-PH

## OBJECTIVE QUESTIONS

1. The energy states for a particle of mass $m$ in a 1-D symmetric infinite potential well of width $2a$ is

$$E_n = \frac{n^2\pi^2\hbar^2}{2m(2a)^2} \quad [n = 1, 2, 3, ...] \quad = \frac{n^2\pi^2\hbar^2}{8ma^2}$$

For ground state, $n = 1$ and for first excited state, $n = 2$

$$\therefore \quad \frac{E_1}{E_2} = \left(\frac{\pi^2\hbar^2}{8ma^2}\right) \Big/ \left(\frac{4\pi^2\hbar^2}{8ma^2}\right) = \frac{1}{4} \quad \Rightarrow \quad E_1 = \frac{E_2}{4}$$

Here, $E_2 = 2\text{ eV}$

$$\therefore \quad E_1 = \frac{2}{4}\text{ eV} = 0.5\text{ eV}$$

**Correct option is (b)**

2. Given the net magnetic moment of iron is $m = 2.2\,\mu_s$

The saturation magnetization is given by = Nm, (where N is the number of atom per unit volume)
For BCC the number of atom per unit volume = $2/a^3$

$$\text{The saturation magnetization is} = \frac{2 \times 2.2 \times 9.273 \times 10^{-24}}{(2.857)^3 \times 10^{-30}}\ \text{Am}^{-1}$$

$$= 1.749 \times 10^6 = 1749\ \text{kAm}^{-1}$$

**Correct option is (d)**

3. Total number of outcomes = $6 \times 6 \times 6 = 216$

Favourable outcomes which have same number in all dice are, (1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5), (6,6,6).

$$\text{Required probability} = \frac{6}{216} = \frac{1}{36}$$

**Correct option is (a)**

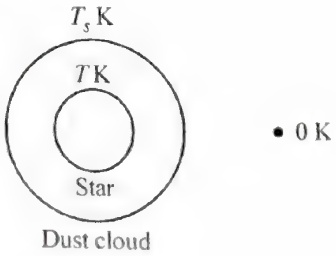
4. Since the dust cloud is at radiative equilibrium, we have

$$(P_{\text{cloud}})_{\text{star}} = (P_{T=0\text{K}})_{\text{cloud}}$$

$$\Rightarrow \sigma\left(T^4 - T_s^4\right) = \sigma\left(T_s^4 - 0^4\right)$$

$$\Rightarrow T_s^4 = \frac{T^4}{2}$$

$$\Rightarrow T_s = 2^{-1/4}T$$




**Correct option is (b)**

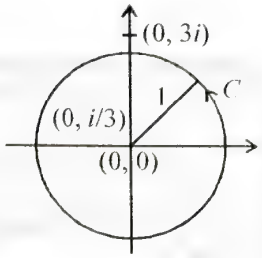
5. $\eta = 1 - \frac{T_c}{T_h} = 1 - \frac{273}{373} \simeq 27\%$

**Correct option is (b)**

7. $$\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)} = \int_0^{2\pi} \frac{d\theta}{5-3\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)} = \int_0^{2\pi} \frac{2i\,d\theta}{10i-3e^{i\theta}+3e^{-i\theta}} \quad \left(\because \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}\right)$$

$$= \int_0^{2\pi} \frac{2ie^{i\theta}\,d\theta}{-3e^{i2\theta}+10ie^{i\theta}+3} = -\frac{2i}{3}\int_0^{2\pi} \frac{e^{i\theta}d\theta}{e^{i2\theta}-\frac{10i}{3}e^{i\theta}-1}$$

Let $z = e^{i\theta} \Rightarrow dz = ie^{i\theta}\,d\theta$




$$I = -\frac{2}{3}\int_C \frac{dz}{\left(z^2 - \frac{10i}{3}z - 1\right)}$$

Condition of Singularity: $z^2 - \frac{10i}{3}z - 1 = 0$

$$\Rightarrow z = \frac{1}{2}\left[\frac{10}{3}i \pm \sqrt{\left(\frac{10i}{3}\right)^2 + 4}\right] \quad \Rightarrow z = \frac{5}{3}i \pm \frac{4}{3}i \text{ or } z = 3i, \frac{i}{3}$$

Only $z = \frac{i}{3}$ lies inside the contour $|z| = 1$.

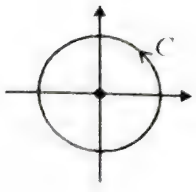
Residue of $f(z)$ at $\left(z = \frac{i}{3}\right) = \underset{z\to\frac{i}{3}}{Lim}\left(z - \frac{i}{3}\right)\frac{1}{(z-3i)\left(z-\frac{i}{3}\right)} = -\frac{3}{8i}$

$$\therefore \quad I = -\frac{2}{3}\int \frac{dz}{(z-3i)\left(z-\frac{1}{3}i\right)} = -\frac{2}{3} \times 2\pi i \times \left[\text{Residue of } f(z) \text{ at } \left(z = \frac{i}{3}\right)\right] = \frac{\pi}{2}$$

**Correct option is (b)**

8. $f(z) = \frac{1}{z}$ has a simple pole at z = 0

Residue of $f(z)$ at $(z = 0) = \underset{z\to 0}{Lim}(z-0)\frac{1}{z} = 1$




Therefor, $\int_C \frac{dz}{z} = 2\pi i \times [\text{Residue of } f(z) \text{ at } (z = 0)] = 2\pi i \times 1 = 2\pi i$

**Correct option is (b)**

9. $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=8e^x$

The homogeneous equation is

$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=0$

The auxilliary equation is $m^2-6m+9=0 \Rightarrow (m-3)^2=0$ or $m=3,3$

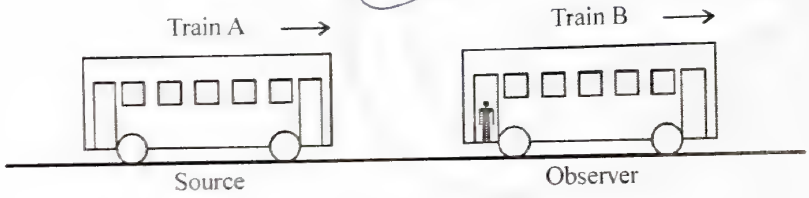
Therefore, complementary function $=Ae^{3x}+Bxe^{3x}$

Particular integral $=\frac{1}{D^2-6D+9}\left(8e^x\right)=8\frac{1}{(1)^2-6.1+9}e^x=2e^x$

Therefore, $y(x)=Ae^{3x}+Bxe^{3x}+2e^x$

**Correct option is (b)**

10. Apparent frequency $f'=\left[\frac{c-v_0}{c-v_s}\right]f_0$




Here, $v_0=10$ m/s; speed of train B

$v_s=$ 43 m/s; speed of train A

$c=$ 330 m/s; speed of sound

$f'=\left(\frac{330-10}{330-43}\right)270 \text{ Hz}=301.04 \text{ Hz}=301 \text{ Hz}$

**Correct option is (a)**

11. Equation of motion for earth, $\frac{mv^2}{R_1}=\frac{GMm}{R_1^2}$

$\Rightarrow v=\sqrt{\frac{GM}{R_1}}$, where $M$ is mass of Sun, $R_1$ is Earth-Sun distance.

Time taken for one revolution,

$T_1=\frac{2\pi R_1}{v}=\frac{2\pi}{\sqrt{GM/R_1}}R_1=\frac{2\pi}{\sqrt{GM}}R_1^{3/2}$ ... (1)

For exoplanet revolving around a star of mass $M/27$, time period

$T_2=\frac{2\pi}{\sqrt{\frac{GM}{27}}}R_2^{3/2}$ ... (2)

Given : $T_1 = T_2$

Dividing Eq. (1) by Eq. (2),

$$\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{GM}} R_1^{3/2} \times \frac{\sqrt{GM}}{2\pi 3^{3/2}} \frac{1}{R_2^{3/2}}$$

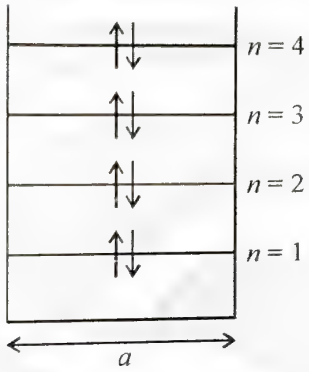
$$\Rightarrow 1 = \left(\frac{R_1}{3R_2}\right)^{3/2} \Rightarrow R_2 = \frac{1}{3} R_1$$

**Correct option is (d)**

12. The $n^{th}$ energy eigenvalue for a particle of mass $m$ in a 1-D box of length $a$ is

$$E_n = \frac{n^2 h^2}{8ma^2} \quad [n = 1, 2, 3, \ldots]$$

The distribution of particles for the ground state will be as shown in the figure:




The minimum total energy

$$= \frac{h^2}{8ma^2}\left(2 \times 1^2 + 2 \times 2^2 + 2 \times 3^2 + 2 \times 4^2\right)$$

$$= \frac{60h^2}{8ma^2} = \frac{15h^2}{2ma^2}$$

**Correct option is (b)**

13. Given : The electric field associated with an EM wave in free space, $\vec{E} = E_m \sin(\omega t - \beta z)\hat{y}$

According to Maxwell's equation : $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\Rightarrow -\frac{\partial \vec{B}}{\partial t} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ 0 & E_m \sin(\omega t - \beta z) & 0 \end{vmatrix}$$

$$\Rightarrow -\frac{d\vec{B}}{dt} = \beta E_m \cos(\omega t - \beta z)\hat{x}$$

$$\Rightarrow \vec{B} = -\frac{\beta E_m}{\omega} \sin(\omega t - \beta z)\hat{x}$$

**Correct option is (b)**

14. When a charge particle moving in a uniform magnetic field, then we can write

$$\frac{mv^2}{R} = qvB$$

$$\Rightarrow R = \left(\frac{mv}{qB}\right)$$

So, $R$ is proportional to $v$ and inversely proportional to $B$.

**Correct option is (c)**

15. The number of spitting of energy level due to presence of magnetic field $= (2l+1)$

For $3p$ $l = 1$

Therefore, the number of spitting $= (2l+1) = (2+1) = 3$

**Correct option is (c)**

16. Given : $V(r) = B\sin 2\alpha r$

$$\vec{F} = -\nabla V$$

$$= -\frac{\partial}{\partial r}(B\sin 2\alpha r)\hat{r}$$

$$= -2B\alpha\cos 2\alpha r\,\hat{r}$$

The force $\vec{F}$ is a central force.

For a central force, the trajectory of the particle is always confined to a plane passing through the origin.

**Correct option is (b)**

17. $$F(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\exp(-ikx)\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\left[\int_{-1}^{1} e^{-ikx} + 0\right] = \frac{1}{\sqrt{2\pi}}\frac{e^{-ik} - e^{ik}}{(-ik)} = \sqrt{\frac{2}{\pi}}\frac{1}{k}\left(\frac{e^{ik} - e^{-ik}}{2i}\right)$$

$$F(k) = \sqrt{\frac{2}{\pi}}\frac{\sin k}{k}$$

**Correct option is (a)**

18. $^{12}C_6 : 1s^2\ 2s^2\ 2p^2$ | ↑ | ↑ | |

$^{16}O_8 : 1s^2\ 2s^2\ 2p^4$ | ↑↓ | ↑ | ↑ |

(i) Larger multiplicity has lower energy.

(ii) For less than half filled lower J value has lower energy.

(iii)For greater than half filled higher J value has lower energy.

For $^{12}C_6 : {}^3P_2 > {}^3P_1 > {}^3P_0$ and $^{16}O_8 : {}^3P_2 < {}^3P_1 < {}^3P_0$

**Correct option is (c)**

19. $F = (A + B \cdot \bar{B})(\bar{A} + B)$

$F = A(\bar{A} + B)$

$F = A \cdot \bar{A} + AB \quad \underbrace{\boxed{x \cdot \bar{x} = 0}}_{\text{Axiom}}$

$F = AB$

**Correct option is (d)**

20. Let the gravitational field at poles of planet be $g$, and that at the equator be $g' = \frac{3}{4}g$

$g' = g - \omega^2 R$

$\Rightarrow \frac{3}{4}g = g - \omega^2 R \Rightarrow g = 4\omega^2 R$

Escape velocity for polar particle,

$v = \sqrt{2gR}$

$= \sqrt{2 \times 4\omega^2 R^2}$

$= \sqrt{8}\omega R = \sqrt{8}\, v_0 \qquad (\because v_0 = \omega R)$

**Correct option is (a)**

21. $f(x) + f\left(x + \frac{1}{2}\right) = 1, \quad 0 \le x \le \frac{1}{2}$

$$\int_0^1 f(x)\,dx = \int_0^{1/2} f(x)\,dx + \int_{1/2}^1 f(x)\,dx$$

$$= \int_0^{1/2} f(x)\,dx + \int_{1/2}^1 \left(1 - f\left(x + \frac{1}{2}\right)\right)dx$$

$$= \int_0^{1/2} f(x)\,dx + \frac{1}{2} - \int_{1/2}^1 f\left(x + \frac{1}{2}\right)dx$$

or $\int_0^1 f(x)\,dx = \int_0^{1/2} f(x)\,dx - \int_0^{1/2} f(t)\,dt + \frac{1}{2} \qquad \left[\text{Taking } x + \frac{1}{2} = t \text{ for the second integral}\right]$

or $\int_0^1 f(x)\,dx = \frac{1}{2} \qquad \left(\text{Since } \int_0^{1/2} f(x)\,dx = \int_0^{1/2} f(t)\,dt\right)$

**Correct option is (c)**

22. It will take time to cross the region $t = (L/u)$

The force acts on the charge particle $F_y = (qE)$

Acceleration $a_y = (qE/m)$

$$\frac{\sqrt{3}}{2}v \times T = \frac{2\pi m}{qB} \times \frac{\sqrt{3}}{2}v = \frac{\sqrt{3}\pi mv}{qB}$$

**Correct option is (b)**

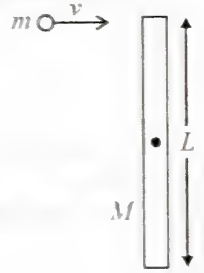
40. Since the mass $m$ is very small compared to mass of rod $M$, so after the mass $m$ sticks to the rod, the centre of mass of the combined system is almost near the centre of mass of the rod. Therefore, we will apply conservation of angular momentum about the centre of mass of the rod.

Applying conservation of linear momentum, $mv = (m+M)v'$

$$v' = \frac{mv}{m+M} = \frac{0.01M}{0.01M+M}v = \frac{v}{101} \quad \text{(Speed of centre of mass)}$$

Applying conservation of angular momentum, $mv\frac{L}{2} = I\omega$

$$\Rightarrow mv\frac{L}{2} = \left(\frac{1}{12}ML^2 + m\left(\frac{L}{2}\right)^2\right)\omega$$




$$\Rightarrow \omega = \frac{mv\frac{L}{2}}{\left(\frac{1}{12}M + \frac{m}{4}\right)L^2} = \frac{mv}{\left(\frac{M}{6} + \frac{m}{2}\right)L}$$

$$= \frac{0.01Mv}{\left(\frac{M}{6} + \frac{0.01M}{2}\right)L} = \frac{0.01v}{\left(\frac{1+0.03}{6}\right)L}$$

$$= \frac{0.06v}{1.03L} = \frac{6v}{103L}$$

$$\omega = \frac{6v}{103L}$$

**Correct option is (b)**

41. The magnetic susceptibility $(\chi)$ of a paramagnetic material varies with temperature as

$$\chi = \frac{C}{T - T_c}, \text{ for } T > T_c$$

**Correct option is (d)**

42. The current density in the semiconductor is given by

$$J = (ne\mu_e + pe\mu_p) = \sigma E$$

$$\Rightarrow \sigma = \left(\frac{ne\mu_e + pe\mu_p}{E}\right) = \frac{10^{14} \times 1.6 \times 10^{-19} \times 5000 + 5 \times 10^{14} \times 1.6 \times 10^{-19} \times 100}{100}$$

$$= \frac{0.8 + 0.032}{100}(\Omega\text{cm})^{-1} = 0.832 \times 10^{-2}(\Omega\text{cm})^{-1} = 8.32\,(\text{m}\Omega\text{cm})^{-1}$$

**Correct option is (d)**

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43. $\Delta W = nRT \ln\left(\frac{V_f}{V_i}\right) = 308\, R \ln\left(\frac{10V_0}{V_0}\right)$

$= 5896.2 \text{ J} \simeq 6 \times 10^3 \text{ J}$

**Correct option is (a)**

44. For adiabatic process, we have

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 293\left(\frac{V_0}{5V_0}\right)^{2/3} = 100.2 \text{ K} \Rightarrow T_f \simeq -173\,^\circ\text{C}$$

**Correct option is (c)**

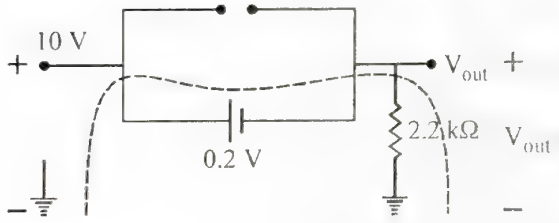
45. The change in entropy is

$$\Delta S = ms \ln\left(\frac{T_f}{T_i}\right) = 5 \times 0.1 \ln\left(\frac{300}{100}\right)$$

$\simeq 0.5$ cal/degree.

**Correct option is (a)**

46. Equivalent circuit:

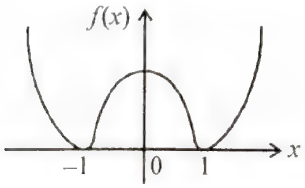



$-10 + 0.2 + V_{out} = 0 \Rightarrow V_{out} = 9.8$ volt

**Correct option is (d)**

47. $f(x) = |x^2 - 1|,\ x \in R$

This function can be plotted as:




$\Rightarrow f(x)$ has local minima at $x = \pm 1$ and local maxima at $x = 0$.

**Correct option is (c)**

48. The electric field at $P(x, y, z)$ is given by

$\vec{E}$ = field at $P$ due to real wire + field at $P$ due to image wire.

$$\Rightarrow \vec{E} = \frac{-\lambda\left(x\hat{i} + (z-d)\hat{k}\right)}{2\pi\varepsilon_0\, x^2 + (z-d)^2} + \frac{\lambda\left(x\hat{i} + (z+d)\hat{k}\right)}{2\pi\varepsilon_0\, x^2 + (z+d)^2}.$$

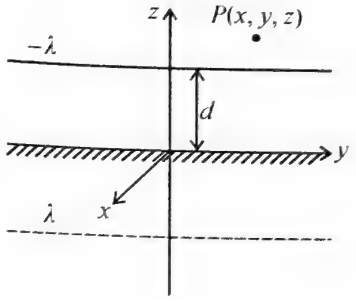
Now the normal component of the electric field on the plane (i.e., at $z = 0$)

$$E_z = \frac{\lambda d}{2\pi\varepsilon_0 x^2 + d^2} + \frac{\lambda d}{2\pi\varepsilon_0 (x^2 + d^2)}$$

$$\Rightarrow \frac{\sigma}{\varepsilon_0} = \frac{\lambda d}{\pi\varepsilon_0 (x^2 + d^2)}$$

$$\Rightarrow \sigma(x) = \frac{\lambda d}{\pi\left(x^2 + d^2\right)}$$

$$\therefore \sigma_{max} = \frac{\lambda d}{\pi d^2} = \frac{\lambda}{\pi d}$$




**Correct option is (a)**

49. If two wave of different frequencies superpose, intensity vary with time period

$$T = \frac{1}{\nu_1 - \nu_2} \Rightarrow \nu_1 - \nu_2 = \frac{1}{T} = \frac{1}{1.5} = \frac{2}{3}\text{ Hz} \quad \ldots (1)$$

Frequency of oscillation is $\frac{\nu_1 + \nu_2}{2}$ $\left[\text{Given : } \frac{\nu_1 + \nu_2}{2} = 256\text{ Hz}\right]$

$$\Rightarrow \nu_1 + \nu_2 = 2 \times 256\text{ Hz} \quad \ldots (2)$$

From Eq. (1) and Eq. (2)

$$\Rightarrow 2\nu_1 = \frac{2}{3} + 2 \times 256 \Rightarrow \nu_1 = \frac{1}{3} + 256 = 256.333\text{ Hz}$$ and $\nu_2$ is $256.333 - \frac{2}{3} = 255.667$ Hz.

**Correct option is (a)**

50. If $\lambda_m$ is wavelength at which intensity of radiation is maximum for a blackbody of temperature $T$, then

$$\lambda_m T = \text{constant}$$

$$\therefore \lambda_{m1} T_1 = \lambda_{m2} T_2$$

$$\lambda_{m2} = \frac{\lambda_{m1}}{T_2} \times T_1 = \frac{600 \times 6000}{300} = 12000\text{ nm} = 12 \times 10^{-6}\text{ m} = 12\ \mu\text{m}$$

**Correct option is (d)**

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Battery removed => charge conf.
Battery connected => potential conf.

# PAPER : DU 2019
## PHYSICS-PH

**INSTRUCTIONS:**

(*i*) This test booklet contains 50 Objective type questions.

(*ii*) Each question carries **4 marks**. **1 mark** will be deducted for each incorrect answer. Only one box should be marked and the marking of more than one box will be treated as wrong answer.

## OBJECTIVE QUESTIONS

1. In 2's complement representation, the number 10110101 represents the decimal number
(a) –53 (b) 75 (c) 93 (d) –75

2. Water has a constant heat capacity of 4.2 J/K/kg over the temperature range 0°–100° C. A kilogram of water, initially at 0°C is brought in contact with a heat reservoir at 100 °C. When the water comes to equilibrium, what is the total change in entropy of the universe?
(a) 2437.8 J/K (b) 1310.8 J/K (c) 184.8 J/K (d) 369.2 J/K

3. The Sun is approximately 25, 000 light years away from the centre of the Milky way and moves around it, in an approximately circular path, in roughly 170 million years. Give that sunlight takes approximately 8 min to reach the Earth, what is the ratio of the masses of the galaxy and the Sun?
(a) $1.5\times10^{12}$ (b) $1.5\times10^{11}$ (c) $5\times10^{11}$ (d) $5\times10^{10}$

4. An AND gate has two inputs A and B and one inhibit input S. The output is 1 if
(a) A = 1, B = 1, S = 1
(b) A = 1, B = 1, S = 0
(c) A = 1, B = 0, S = 0
(d) A = 1, B = 0, S = 1

5. A wire carrying a current 2A is bent into the shape of a two semi circles, one with a radius 1m and the other with radius 2m. The two semi circles are connected at their ends that is along the two radii. The magnetic field at the centre of the semi-circle is
(a) $\pi\times10^{-7}$ T
(b) cannot be determined from the information provided above.
(c) $16\times10^{-7}$ T
(d) $2\times10^{-7}$ T

CAREER ENDEAVOUR

6. Under ideal conditions, the electric and magnetic fields inside a superconductor are zero. Which of the following must be true just outside the surface of the superconductor?
(a) B = 0
(b) B is perpendicular to the surface
(c) B is tangential to the surface
(d) The magnetic flux is quantized

7. Under certain conditions, a beam of electrons impinging on a crystal surface will diffract and a scattering pattern of the beam can be obtained. What is the approximate kinetic energy of the electrons needed in order to see the pattern? (Assume the lattice spacing of the crystal to be 0.4 nm)
(a) 100 eV (b) 1 eV (c) 0.1 eV (d) 10 eV

8. Fermat's principle of ray optics states, "A ray of light follows the path between two points which requires the least time." This principle can be used to derive which of the following?
(I) Snell's law of refraction
(II) The law of reflection
(III) Rayleigh's criterion for resolution
(a) I and II (b) I only (c) I, II and III (d) II only

9. For a mole of ideal gas at T = 35 °C, what is the work done for an isothermal expansion from a volume $V_0$ to $10V_0$?
(a) $10^4$ J (b) $10^3$ J (c) $3\times10^3$ J (d) $6\times10^3$ J

10. A single photon of energy 100 MeV decays into an electron and a positron. Which of the following statements is true?
(a) It does not violate any of the above conservation laws.
(b) This process violates energy momentum conservation.
(c) This process violates angular momentum conservation
(d) This process violates lepton number conservation.

11. An electric field $E$ can pull an electron out of a Helium atom in the same time that the electron takes to go around the nucleus once. Then $E$ is
(a) $2\times10^{12}$ V/m (b) $5\times10^{11}$ V/m (c) $10^{12}$ V/m (d) $10^{13}$ V/m

12. A material has a heat capacity at constant volume C that is unchanging over a wide temperature range. Two identical bodies made of this material, at initial temperature T and 9T, respectively, are used to drive a heat engine. The maximum amount of work that can be obtained from this engine is
(a) 9 CT (b) 4 CT (c) 8 CT (d) 3 CT

13. An electrical device operates on 9V and has a resistance of 21Ω. It is connected to a power supply of 120 V output through a transformer. The current in the primary coil of the transformer is
(a) 0.032 A (b) Cannot be determined from the information supplied.
(c) 0.23 A (d) 2.32 A

14. The frequency of oscillation of an LC circuit is ω. The plates of the parallel plate capacitor are pulled apart to twice the original distance and a dielectric constant $k$ is inserted. The frequency of oscillation for the new configuration is
(a) $\omega\sqrt{2/k}$ (b) $k\omega/2$ (c) $2\omega/k$ (d) $\omega\sqrt{2/k}$

15. One mole of a monoatomic perfect gas is initially at a constant temperature $T_0$. It expands from a volume $V_0$ to $2V_0$ under constant pressure. The heat absorbed by the gas is
(a) $\frac{3}{2}RT_0$ (b) $\frac{5}{2}RT_0$ (c) $2RT_0$ (d) $RT_0$

16. A scalar meson decays into three pseudoscalar mesons. The decay happens through
(a) strong interaction (b) weak interaction
(c) gravitational interaction (d) electromagnetic interaction

17. An ideal monoatomic gas, initially at T = 20 °C, expands adiabatically from a volume $V_0$ to $5V_0$. What is the final temperature ?
(a) –33 °C (b) –113 °C (c) –20 °C (d) –173 °C

18. Consider three identical, ideal capacitors. The first capacitor is charged to a voltage V and then disconnected from the battery. The other two capacitors, initially uncharged and connected in series, are then connected across the first. What is the final voltage across the first capacitor?
(a) V/3 (b) V (c) 2V/5 (d) 2V/3

19. Two rockets approach a planet from opposite directions with speed $\pm\frac{c}{2}$ with respect to the planet. The proper length of the each rocket is $L$. How long does each rocket appear to the other rocket?
(a) $\frac{\sqrt{3}}{5}L$ (b) $\frac{3}{5}L$ (c) $\frac{\sqrt{3}}{2}L$ (d) $\frac{1}{3}L$

20. Consider the $(2\pi)$-periodic function $f(x)$ defined by

$$f(x)=\begin{cases} x(\pi-x), & x\in[0,\pi] \\ x(x-\pi), & x\in[-\pi,0] \end{cases}$$

Which of the following is true?

(a) $f(x)=\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{\sin([2k+1]x)}{(2k+1)^2}+\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{\cos([2k+1]x)}{k^2}$

(b) $f(x)=\frac{8}{\pi}\sum_{k=0}^{\infty}\frac{\sin(kx)}{k^3}$

(c) $f(x)=\frac{8}{\pi}\sum_{k=0}^{\infty}\frac{\sin[(2k+1)x]}{(2k+1)^3}$

(d) $f(x)=\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{\sin(kx)}{k^2}+\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{\cos(kx)}{k^2}$

21. Consider a rotating spherical planet such that the effective gravitational attraction at the equator is only 75% of that at the pole. If the linear velocity of a point on the equator is $v_0$, what is the escape velocity for a polar particle?

(a) $v_0\sqrt{2}$ (b) $2v_0$ (c) $v_0\sqrt{8}$ (d) $4v_0$

22. If the Fourier transfor of $e^{-ax^2}$ is $e^{-k^2/4a}/\sqrt{a}$, then the Fourier transform of $f(x)=x^2e^{-ax^2}$ is

(a) $F(k)=\frac{k^2-2a}{4a^{5/2}}e^{-k^2/4a}$ (b) $F(k)=\frac{-k^2+2a}{4a^{5/2}}e^{-k^2/4a}$

(c) $F(k)=\frac{k^2+2a}{4a^{3/2}}e^{-k^2/4a}$ (d) $F(k)=\frac{k^2+2a}{4a^{3/2}}e^{-k^2/4a}$

23. Let C be the unit circle, travelled counterclockwise. Evaluate

$$\oint_C\left[\left(e^{-x^2}-y^3\right)dx+x^3dy\right]$$

(a) $2\pi$ (b) $\frac{3\pi}{2}$ (c) $\frac{\sqrt{\pi}}{2}+2$ (d) $\frac{\sqrt{\pi}}{2}-2$

24. A body of mass m moving with a speed $v$ collides with an identical body at rest and sticks with it. The mass M and momentum $P$ of the final clump of matter is $\left(\gamma=\frac{1}{\sqrt{1-v^2/c^2}}\right)$

(a) $M=2m,\ P=2\gamma mv$ (b) $M=m\gamma,\ P=\gamma mv$

(c) $M=m\sqrt{\gamma},\ P=mv/2$ (d) $M=m\sqrt{(2(1+\gamma))},\ P=\gamma mv$

25. Using Boolean algebra, simplifying the expression,

$$\overline{\overline{(a\cdot b\cdot \overline{c})}+\overline{(\overline{c}\cdot d)}}$$

yields

(a) $a\cdot b+\overline{c}\cdot d$ (b) $a\cdot b\cdot d$ (c) $a\cdot b+d$ (d) $a\cdot b\cdot \overline{c}\cdot d$

26. A large and spherical soap film of thickness $d$ has a refractive index 4/3. A narrow beam of yellow light $(\lambda \approx 6400\text{Å})$ is incident on the film at an angle of 30°. What is the value of $d$ for which a constructive second order interference would occur for the reflected wave?

(a) 4800 Å (b) 5500Å (c) 5200Å (d) 3900Å

27. The integral,

$$\oint_{-\infty}^{\infty} \frac{d\delta(y)}{dy}\sin y\,dy$$

is.

(a) $\pi$ (b) 0 (c) +1 (d) −1

28. A cylinder of length $L$ is made up of an inner core of steel of radius $r$ and an outer sheath of copper of thickness $r$. The resistivities of steel and copper are $\rho_1$ and $\rho_2$ respectively. The total resistance of the cylinder is

(a) $\dfrac{(3\rho_1+\rho_2)L}{\pi r^2}$ (b) $\dfrac{\rho_1\rho_2 L}{\pi r^2(\rho_1+\rho_2)}$ (c) $\dfrac{(2\rho_1+\rho_2)L}{\pi r^2}$ (d) $\dfrac{\rho_1\rho_2 L}{\pi r^2(3\rho_1+\rho_2)}$

29. A particle confined to move in $0\le x\le\infty$ is described by the wave function $\psi(x)=e^{-x/2}$. Calculate the probability of finding the particle between $x=0$ and $x=\ln(2)$

(a) 2 (b) 1/2 (c) 1 (d) 3/2

30. Let $V$ be the region in $R^3$ determined by the inequalities $x^2+y^2\le 1$ and $0\le z\le 4-x^2-y^2$. Determine

$$\iiint_V (x^2+y^2)\,d^3r$$

(a) $\dfrac{5\pi}{3}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{2\pi}{3}$ (d) $\dfrac{5\pi}{4}$

31. Let $\psi_0(x)$ and $\psi_1(x)$ be the wave functions corresponding to the ground and the first excited states of a one dimensional harmonic oscillator respectively. Consider the normalized state $\phi(x)=\alpha\psi_0(x)+\beta\psi_1(x)$ where $\alpha$ and $\beta$ are real numbers. The values of $\alpha$ and $\beta$ for which, $\langle x\rangle$, the average value of the position is a minimum are

(a) $\alpha=-\beta=\dfrac{1}{\sqrt{2}}$ (b) $\alpha=\beta=\dfrac{1}{\sqrt{2}}$

(c) $\alpha=\dfrac{1}{\sqrt{3}}$ and $\beta=-\sqrt{2/3}$ (d) $\alpha=\dfrac{1}{\sqrt{3}}$ and $\beta=\sqrt{2/3}$

32. A particle confined in one dimension, due to potential $V(x)$, has the following wave function:

$$\psi(x) = \left(\frac{x}{a}\right)^n \exp\left(-\frac{x}{a}\right)$$

where '$a$' is a constant. If $V(x) \to 0$ as $x \to \infty$, then $V(x)$ is given by

(a) $\frac{\hbar^2}{2m}\left[\frac{n(n+1)}{x^2} - \frac{n}{ax}\right]$ (b) $\frac{\hbar^2}{2m}\left[\frac{n(n-1)}{x^2} + \frac{n+1}{ax}\right]$

(c) $\frac{\hbar^2}{2m}\left[\frac{n(n+1)}{x^2} + \frac{n-1}{ax}\right]$ (d) $\frac{\hbar^2}{2m}\left[\frac{n(n-1)}{x^2} - \frac{2n}{ax}\right]$

33. Let $S$ be the surface defined by the parametric equations $x = 4\alpha^2 + \beta$, $y = \sin\beta$, $z = \cos\alpha$. The equation for the plane tangent to $S$ at the point $(\pi^2, 0, 0)$ is given by

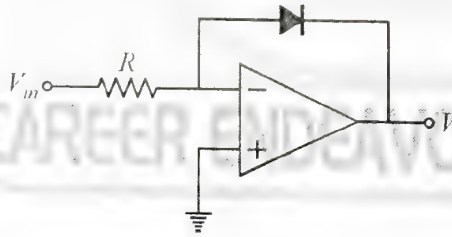
(a) $x + y - 2\pi z = \pi^2$ (b) $4x + y - z = 4\pi^2$ (c) $4x - y + z = 4\pi^2$ (d) $x - y + 4\pi z = \pi^2$

34. For waves in a certain medium, the group velocity is $v_g$ while the phase velocity is $v_p$. Which of the following is necessarily true ? ($c$ is the velocity of light).

(a) $v_p < c$ and $v_g$ can be anything (b) $v_g < c$ and $v_p$ can be anything

(c) $v_p < v_g < c$ (d) $v_g < c$ and $v_g v_p < c^2$

35. The circuit of following figure uses an ideal op-amp for small positive values of $V_{in}$. The circuit works as:




(a) an exponential amplifier (b) a half wave rectifier

(c) a logarithmic amplifier (d) a differentiator

36. For $-1 \le x \le +1$, the sum $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$ equals

(a) $\sin^2 x$ (b) $\frac{x \exp x}{\pi}$ (c) $\cos^2 x$ (d) $\tan^{-1} x$

37. A narrow light beam of wavelength $\lambda$ is travelling along the $x$-axis, when it is faced by a barrier with narrow slits at $y = 0, \pm d$ (and all at $x = 0$). There is a vertical screen at $x = D \gg d$. What is the angle $\theta$ that the central bright spot and the next-nearest dark spot subtend at the central slit $[(x, y) = (0, 0)]$

(a) $\theta = \sin^{-1}\frac{\lambda}{2d}$ (b) $\theta = \sin^{-1}\frac{2\lambda}{d}$ (c) $\theta = \sin^{-1}\frac{3\lambda}{d}$ (d) $\theta = \sin^{-1}\frac{\lambda}{3d}$

38. A stream of water flowing with a constant speed $v$ manages to move some pebbles. If the density of water is $\rho$ and the acceleration due to gravity is $g$, then the mass of the heaviest pebble that it can move is (here, $A$ is a constant)

(a) $\frac{A\rho v^5}{g^2}$ (b) $\frac{A\rho v^6}{g^3}$ (c) $\frac{A\rho^2 v^3}{g^4}$ (d) $\frac{A\rho^2 v^8}{g^4}$

39. The expectation value of momentum of a particle whose wave function is:

$\Psi(x) = A\exp\left(-\frac{x^2}{2a^2} + ikx\right)$ is given as:

(a) $\frac{ih}{a^2}$ (b) $\hbar k$ (c) 0 (d) $\frac{\hbar k}{a}$

40. Two particles of mass $m$ each are connected by a massless spring of length $\ell$ and spring constant $k$. The system is lying (in the relaxed state) on a frictionless horizontal table. A very short impulse $P$ is applied to the first mass (along the axis of the system) at $t = 0$. When would the *second* mass first come to rest?

(a) $t = \pi\frac{\sqrt{m}}{2k}$ (b) $t = 2\pi\frac{\sqrt{2m}}{k}$ (c) $t = \pi\frac{\sqrt{m}}{k}$ (d) $t = \pi\frac{\sqrt{2m}}{k}$

41. Consider a particle whose normalized wave function is:

$$\psi(x) = \begin{cases} 2\alpha\sqrt{\alpha}xe^{-\alpha x} & ; \quad x > 0 \\ 0 & ; \quad x < 0 \end{cases}$$

For what value of $x$ does $P(x) = |\psi(x)|^2$ peak ?

(a) $x = 0$ (b) None of these (c) $x = \frac{1}{\alpha}$ (d) $x = 1$

42. A self-contained machine has, as its input, two equal steady stream of a fluid at temperature $T_1$ and $4T_1$ respectively. The only output is a single high-speed jet of the same fluid at temperature $T$. The heat capacity per unit mass of the fluid is $C$ (and is independent of the temperature). What is the maximum possible speed of the jet ?

(a) $\sqrt{CT_1}$ (b) $\sqrt{\frac{CT_1}{4}}$ (c) $\sqrt{2CT_1}$ (d) $\sqrt{4CT_1}$

43. A particle of mass $M$ decays at rest into two particles. One particle has mass $m$ and the other particle is massless. The momentum of the massless particle is

(a) $\frac{M^2 - 2m^2}{4M}c$ (b) $\frac{M^2 - m^2}{2M}c$ (c) $\frac{M^2 + m^2}{2M}c$ (d) $\frac{M^2 - m^2}{4M}c$

44. The distribution of weights of 100 students in a class follows normal distribution with a mean of 58 kg and a standard deviation of 25 kg. How many students have weights in the range 53 – 63 kg ?

(a) 32 (b) 68 (c) 99 (d) 88

45. A gas cell with an optical path length of 10 cm is placed in one arm of a Michelson interferometer. If the light source for the interferometer is a laser with a wavelength of 632.2 nm, then 100 fringes are counted as the gas cell is evacuated. What is the index of refraction of the original gas ?

(a) 0.99968 (b) 1.00063 (c) 1.00032 (d) 1.00016

46. A child of mass 3m sits on a swing, the basis of which has mass $m$, while the rope (of length $l$) has a negligible mass. An older child pulls back the swing (with the child) until the rope makes an angle of half-a-radian with the vertical, and, then, pushes with a force mg along the arc of a circle untill the rope is exactly vertical, and releases it. If the rope was always taut, for how long did the child push ?

(a) $\pi\dfrac{\sqrt{2\ell}}{3g}$ (b) $\dfrac{\sqrt{2\ell}}{3g}$ (c) $\dfrac{\pi}{3}\dfrac{\sqrt{\ell}}{g}$ (d) $\dfrac{\sqrt{\ell}}{g}$

47. X-rays undergo Bragg reflection from a simple cubic crystal, with a density known with an accuracy of one percent. If the angle of the reflected X-ray with the crystal plane is 6° and is measured with an accuracy of 3 minutes of arc, what is the relative error in the measurement of wavelength?

(a) $5.6 \times 10^{-3}$ (b) $1.1 \times 10^{-2}$ (c) $3.3 \times 10^{-3}$ (d) $6.5 \times 10^{-3}$

48. An uncharged spin-half particle of mass $m$ is confined to move on a circular wire of radius $a$. The particle has magnetic moment $\mu$. A magnetic field $B$ is applied perpendicular to the plane of the wire. Then the energy eigenvalues of the particle are (here $n$ is an integer)

(a) $\dfrac{\hbar^2 n^2}{4ma^2} \pm 2\mu B$ (b) $\dfrac{\hbar^2 n^2}{4ma^2} \pm \mu B$ (c) $\dfrac{\hbar^2 n^2}{2ma^2} \pm 2\mu B$ (d) $\dfrac{\hbar^2 n^2}{2ma^2} \pm \mu B$

49. A photon of initial wavelength 0.4 Å suffers two successive collision with two electrons. The deflection in the first collision is 90° and in the second collision is 60°. The final wavelength of the photon is

(a) 0.024 Å (b) 0.436 Å (c) 0.4 Å (d) 0.012 Å

50. A sample of a certain element is placed in a 0.300-T magnetic field. How far apart are the Zeeman components of the 450-nm spectral line of this element ?

(a) 0.000283 nm (b) 2.83 nm (c) 0.00283 nm (d) 0.23 nm

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# SOLVED PAPER : DU 2019

## PHYSICS-PH

## OBJECTIVE QUESTIONS

1. 2's complement of 10110101 = 1's complement of 10110101 + 1 = 01001010 + 1

sign bit ←
0→+ve
1→ –ve

$= \boxed{0}\ 1001011$

$= \left(1\times2^6+0\times2^5+0\times2^4+1\times2^3+0\times2^2+1\times2^1+1\times2^0\right) = (64+8+2+1) = 75$

**Correct option is (b)**

2. The amount of heat exchanged between water and reservoir is

$Q = ms\Delta T$

$= 1\times4.2\times(373-273) = 420\ kJ$

Change of entropy of water is

$$\Delta S_w = ms\ln\left(\frac{373}{273}\right) = 1\times4.2\times\ln\left(\frac{373}{273}\right) = 1.31084\ \text{kJ/K}$$

And change of entropy of reservoir is

$$\Delta S_R = -\frac{Q}{373} = -\frac{420}{373} = -1.1260\ \text{kJ/K}$$

Therefore, change of entropy of the universe is

$\Delta S_{universe} = \Delta S_W + \Delta S_R = 184.82\ \text{J/K}$

**Correct option is (c)**

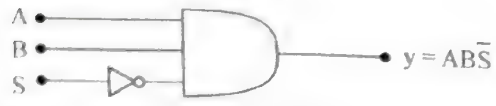
3. According to Kepler law,

$$T_{Sun} = \frac{2\pi}{\sqrt{GM_{milky}}}\times r_s^{3/2} \text{ and } T_{Earth} = \frac{2\pi}{\sqrt{GM_{Sun}}} r_e^{3/2}$$

$$\therefore \quad \frac{T_{Sun}}{T_{Earth}} = \left(\frac{r_s}{r_e}\right)^{3/2}\times\left(\sqrt{\frac{M_{Sun}}{M_{milky}}}\right)$$

$$\rightarrow \quad \frac{M_{milky}}{M_{Sun}} = \frac{T_{Earth}^2}{T_{Sun}^2}\times\frac{r_s^3}{r_e^3} = \left(\frac{1}{170\times10^6}\right)^2\times\left(\frac{25000\times365\times24\times60}{8}\right)^3 = 1.5\times10^{11}$$

**Correct option is (b)**

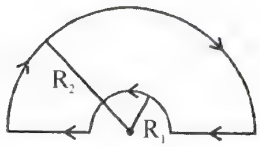
4. 


When, A = 1 , B = 1, S = 0, y = 1

**Correct option is (b)**

5. The magnetic field at the centre of the semi circle is,

$$B = \left(\frac{\mu_0 I}{4R_1} - \frac{\mu_0 I}{4R_2}\right) = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$




$$= \frac{4\pi \times 10^{-7} \times 2}{4} \times \left(\frac{1}{1} - \frac{1}{2}\right) = \pi \times 10^{-7}\ T$$

**Correct option is (a)**

6. According to Boundary condition of magnetostatics

$$(B_{in})_n = (B_{out})_n$$

$$\Rightarrow \quad (B_{out})_n = 0 \qquad (\text{As } B_{in} = 0)$$

And $\quad (H_{out})_\tau - (H_{in})_\tau = \vec{k}$

$$\Rightarrow \quad (H_{out})_\tau = \vec{k} \neq 0 \quad (H_{in} = 0)$$

$$\Rightarrow \quad (\vec{B}_{out})_\tau \neq 0$$

Therefore, the magnetic field just out side the super conductor will be tangential to the surface.

**Correct option is (c)**

7. Lattice spacing of the crystal $\simeq 0.4$ nm

For the electron diffraction, $\lambda \sim d$

$$\Rightarrow \quad 0.4\ nm = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow \quad \left(0.4 \times 10^{-9}\right)^2 = \frac{\left(6.6 \times 10^{-34}\right)^2}{2 \times 9.11 \times 10^{-31} \times E}$$

$$\Rightarrow \quad E = 14.95 \times 10^{-68+31+18}\ \text{Joule} = \frac{14.94237 \times 10^{-68+18+31}}{1.6 \times 10^{-19}}\ \text{eV}$$

$$= 9.34\ eV \cong 10\ \text{eV}$$

**Correct option is (d)**

8. Fermat's principle can be used to derive Snell's law and the law of reflection

**Correct option is (a)**

9. Work done for an isothermal expansion is given by

$$\Delta W = nRT \ln\left(\frac{V_f}{V_i}\right) = 8.314 \times 308 \times \ln\left(\frac{10\,V_0}{V_0}\right) = 5896.25\ J \simeq 6 \times 10^3\ J$$

**Correct option is (d)**

10. Photon $\longrightarrow$ electron + position

$$\gamma = e^- + e^+$$

(i) Lepton number (L): $0 = (+1) + (-1) \Rightarrow \Delta L = 0$

Lepton number is conserved.

(ii) Energy-momentum conservation:

**Energy conservation:**

$$h\nu = m_e c^2 + m_e c^2 + k_{e^-} + k_{e^+}$$

$$\Rightarrow \quad 100\ MeV = (0.51 + 0.51) + 2k_e \text{ (Let kinetic energy of electron and positron are same)}$$

$$\Rightarrow \quad 2k_e = (100 - 1.02)\ MeV$$

$$\Rightarrow \quad 2k_e = 98.98\ MeV$$

$$\Rightarrow \quad \boxed{k_e = 49.49\ MeV}$$

Hence, energy momentum conservation holds.

**Linear momentum:**

$$p_\gamma = p_e + p_{e^-}$$

$$\Rightarrow \quad \boxed{\frac{h\nu}{c} = p_e + p_{e^-}} \Rightarrow \boxed{\frac{h\nu}{2c} = p_e} \text{ (if we consider momentum of electron and positron equal).}$$

**(iii) Angular momentum conservation:**

Let us check spin and momentum:

$$0 \rightarrow \frac{1}{2} \oplus \frac{1}{2} \Rightarrow \boxed{\Delta S = 0}$$

Thus, spin angular momentum is conserved. Since, orbital angular momentum is measured by integral ($l$), 0, 1, 2, 3, .....

Therefore, the total angular momentum is also conserved.

Hence, we can say the process

$$\boxed{\gamma \rightarrow e^+ + e^-}$$

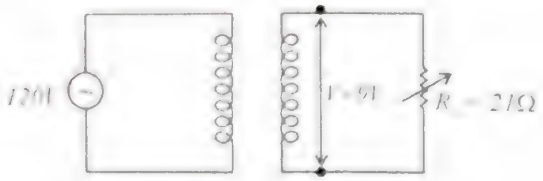
Thus, not violate any of the above conservation laws.

**Correct option is (a)**

12. The maximum work is

$$\Delta W = C\left(\sqrt{T_1} - \sqrt{T_2}\right)^2 = C\left(\sqrt{9T} - \sqrt{T}\right)^2 = 4CT$$

**Correct option is (b)**

13. 


The current in the secondary circuit is

$$I_s = \left(\frac{V}{R}\right) = \left(\frac{9}{21}\right) A = \left(\frac{3}{7}\right) A$$

According to energy consrevation

$$P_P = P_S$$

$$\Rightarrow \quad I_P V_P = I_S V_S$$

$$\Rightarrow \quad I_P \times 120 = \frac{3}{7} \times 9$$

$$\Rightarrow \quad I_P = \left(\frac{27}{7 \times 120}\right) = 0.0321\ A$$

**Correct option is (a)**

14. The frequency of LC circuit is given by $\omega = \frac{1}{\sqrt{LC}}$

The capacitance of a parallel plate capacitor is

$$C = \left(\frac{\varepsilon_0 A}{d}\right)$$

Therefore, the capacitance of the new parallel plate capacitor is

$$C' = \frac{k\varepsilon_0 A}{2d} = \left(\frac{k}{2}C\right)$$

Therefore, the frequency of the new LC circuit is

$$\omega' = \frac{1}{\sqrt{LC'}} = \frac{1}{\sqrt{L\frac{k}{2}C}} = \omega\sqrt{\frac{2}{k}}$$

**Correct option is (a)**

15. At constant pressure, we have

$$\frac{T_i}{T_i} = \frac{V_f}{V_i} \Rightarrow T_f = T_0\left(\frac{2V_0}{V_0}\right) = 2T_0$$

$$\Delta Q = nC_p\Delta T = \frac{5}{2}R(2T_0 - T_0) = \frac{5}{2}RT_0$$

**Correct option is (b)**

16. Scalar meson $\rightarrow$ three pseudoscalar mesons

We can take one example of this type of decay:

$$K^+ \longrightarrow \pi^+ + \pi^+ + \pi^-$$

Charge $(Q)$: $\quad +1 \quad (+1)\ (+1)\ (-1) \quad \Rightarrow \Delta Q = 0$

Strangeness $(S)$: $\quad +1 \longrightarrow (0)\ (0)\ (0) \quad \Rightarrow \Delta S = 0$

Since, strangeness is not conserved in this decay mode.

Therefore, we can say, this decay mode is possible via weak interaction.

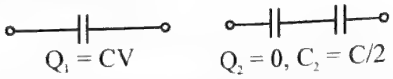
**Correct option is (b)**

17. For adiabatic process, we have

$$T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i = \left(\frac{V_0}{5V_0}\right)^{2/3} \times 293 = 100.2\ K = -172.79\,^\circ C \simeq -173\,^\circ C$$

**Correct option is (d)**

18. Before connection ,

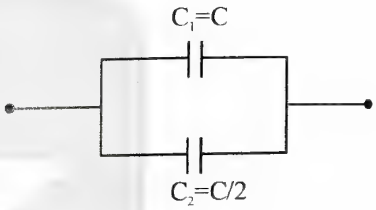



After connecting them in parallel, some charge will flow from $C_1$ to $C_2$ untill the potential difference become equal.

$$\therefore \quad \frac{Q_1 - Q'}{C} = \frac{Q'}{C/2}$$

$$\Rightarrow \quad 3Q' = Q_1$$

$$\Rightarrow \quad Q' = \left(\frac{Q_1}{3}\right) = \left(\frac{CV}{3}\right)$$



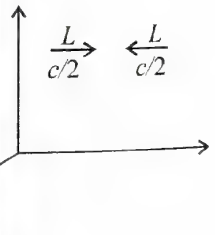

Therefore, the final voltage across the first capacitor is

$$V_1 = (Q_1 - Q_1/3)/C_1$$

$$= \frac{2Q_1/3}{C_1} = \frac{2CV}{3C} = \left(\frac{2V}{3}\right)$$

**Correct option is (d)**

19. The velocity of one rocket with respect to other rocket will be




$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{\frac{c}{2} + \frac{c}{2}}{1 + \frac{c^2}{4c^2}} = \frac{4c}{5}$$

Therefore, the length of the rocket with respect to the other rocket

$$= L\sqrt{1 - \left(\frac{4c}{5}\right)^2/c^2} = L\sqrt{1 - \frac{16}{25}} = \frac{3}{5}L$$

**Correct option is (b)**

20. $$f(x) = \begin{cases} x(\pi - x) & x \in [0, \pi] \\ x(x - \pi) & x \in [-\pi, 0] \end{cases}$$

General form of the Fourier series of the given function, is

$$f(x) = a_0 + \sum_{k=1}^{\infty} [a_k \cos(kx) + b_k \sin(kx)]$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(x)\,dx = \frac{1}{2\pi}\left[\int_{-\pi}^{0} x(x-\pi)\,dx + \int_{0}^{\pi} x(\pi - x)\,dx\right]$$

$$= \frac{1}{2\pi}\left[\left\{-\pi\frac{x^2}{2}+\frac{x^3}{3}\right\}\Big|_{-\pi}^{0}+\left\{\pi\frac{x^2}{2}-\frac{x^3}{3}\right\}\Big|_{0}^{\pi}\right] = \frac{1}{2\pi}\left[\left\{\frac{\pi^3}{2}+\frac{\pi^3}{3}\right\}+\left\{\frac{\pi^3}{2}-\frac{\pi^3}{3}\right\}\right] = \frac{1}{2}\pi^2$$

None of the given options have a constant term of $\pi^2/2$.

**Therefore, none of the options is correct.**

21. Let the gravitational field at poles of planet be $g$, and that at the equator be $g' = \frac{3}{4}g$

$$g' = g - \omega^2 R$$

$$\Rightarrow \frac{3}{4}g = g - \omega^2 R \Rightarrow g = 4\omega^2 R$$

Escape velocity for polar particle,

$$v = \sqrt{2gR}$$

$$= \sqrt{2 \times 4\omega^2 R^2}$$

$$= \sqrt{8}\omega R = \sqrt{8}\, v_0 \qquad (\because\ v_0 = \omega R)$$

**Correct option is (c)**

22. Fourier transform of $e^{-ax^2}$ is

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ax^2} e^{ikx}\,dx = \frac{e^{-k^2/4a}}{\sqrt{a}} \qquad \text{(1)}$$

Differentiating both sides of equation (1) with respect to '$a$', we have,

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(-x^2)e^{-ax^2}e^{ikx}\,dx = e^{-k^2/4a}\left(-\frac{1}{2\sqrt{a}}\right)+\frac{1}{\sqrt{a}}e^{-k^2/4a}\left(-\frac{k^2}{4}\right)\left(-\frac{1}{a^2}\right) = \frac{k^2-2a}{4a^{5/2}}e^{-k^2/4a}$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} x^2\exp(-ax^2)e^{-ikx}\,dx = \frac{-k^2+2a}{4a^{5/2}}e^{-k^2/4a}$$

Therefore, the Fourier transform of $f(x) = x^2 e^{-ax^2}$ is $\frac{-k^2+2a}{4a^{5/2}}e^{-k^2/4a}$

**Correct option is (b)**

23. Given : $I = \oint_C\left[\left(e^{-x^2}-y^3\right)dx + x^3\,dy\right]$, (where $C : x^2+y^2 = 1$)

According to Stokes theorem, $\oint_C \vec{F}\cdot d\vec{l} = \int_S \left(\vec{\nabla}\times\vec{F}\right)\cdot d\vec{S}$

Here, $\vec{F} = \left(e^{-x^2}-y^3\right)\hat{i} + x^3\hat{j}$

Therefore, $\vec{\nabla}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ e^{-x^2}-y^3 & x^3 & 0\end{vmatrix}=\hat{k}\left[3x^3-\left(-3y^2\right)\right]=3\left(x^2+y^2\right)\hat{k}=3r^2\,\hat{k}$

Therefore, $I=3\int_S r^2\,\hat{k}\cdot\left(r\,dr\,d\theta\right)\hat{k}=3\int_0^{2\pi}d\theta\int_0^1 r^3\,dr=3\times 2\pi\times\left.\frac{r^4}{4}\right|_0^1=\frac{3\pi}{2}$

**Correct option is (b)**

24. Let $p$ and $P$ respectively denote the momenta of $m$ and $M$.

If $v'$ is the speed of $M$, then according to momentum conservation,

$$\frac{mv}{\sqrt{1-v^2/c^2}}=\frac{Mv'}{\sqrt{1-v'^2/c^2}}$$

$$\Rightarrow \frac{mv}{\sqrt{1-v^2/c^2}}=\gamma mv=p$$

Now, according to energy conservation,

$$\sqrt{p^2c^2+m^2c^4}+mc^2=\sqrt{P^2c^2+M^2c^4}$$

Squaring both side, we get

$$p^2c^2+m^2c^4+m^2c^4+2mc^2\sqrt{p^2c^2+m^2c^4}=P^2c^2+M^2c^4$$

$$\Rightarrow \quad 2m^2c^4+2mc^2\left(mc^2\gamma\right)=M^2c^4 \qquad \left(\because\ P=p \text{ and } \gamma mc^2=\sqrt{p^2c^2+m^2c^4}\right)$$

$$\Rightarrow \quad M^2=m^2\left(2\left(1+\gamma\right)\right)$$

$$\Rightarrow \quad M=m\sqrt{2\left(1+\gamma\right)}$$

**Correct option is (d)**

25. $y=\overline{\left(a\cdot b\cdot\bar{c}\right)}+\overline{\bar{c}\cdot d}=\left(\overline{\overline{ab\bar{c}}}\bullet\overline{\overline{\bar{c}d}}\right)=ab\bar{c}d$

**Correct option is (d)**

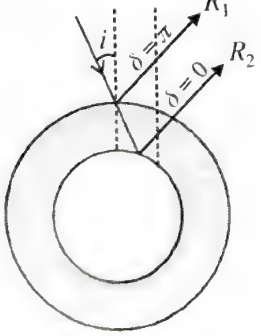
26. Path difference between rays $R_1$ and $R_2$ is $\Delta=2nd\cos r$.

Here, additional phase change is $\pi$, therefore, for constructive interference

$$2nd\cos r=\left(m+\frac{1}{2}\right)\lambda;\ \text{if } m=0,1,2,3,\ldots$$

$$=\left(m-\frac{1}{2}\right)\lambda;\ \text{if } m=1,2,3,4,\ldots$$

$$\Rightarrow d=\frac{\left(m\pm\frac{1}{2}\right)\lambda}{2n\cos r}$$




Here, $\lambda=6400$ Å, $n=\frac{4}{3}$

Using Snell's law, $n_1 \sin i = n_2 \sin r$

$$\Rightarrow \sin r = \frac{n_2}{n_1}\sin i = \frac{4}{3}\times\frac{1}{2} = \frac{4}{6} \Rightarrow r = 41.81^\circ$$

For second order $m = 2$

$$d = \frac{\left(m \pm \frac{1}{2}\right)\lambda}{2n\cos r} = \frac{\left(2 \pm \frac{1}{2}\right)6400\,\text{Å}}{2\times\frac{4}{3}\cos 41.8^\circ} = 4829.12\,\text{Å or } 11267.96\,\text{Å}$$

**Correct option is (a)**

27. We know that, $\int_{-\infty}^{\infty} f(y)\,\delta^{(n)}(y-a)\,dy = (-1)^n f^{(n)}(a)$

Here, $\int_{-\infty}^{\infty} \frac{d\delta(y)}{dy}\sin y\,dy = \int_{-\infty}^{\infty} \sin y\,\delta^{(1)}(y-0)\,dy = (-1)^1 \frac{d}{dy}(\sin y)\Big|_{y=0} = -1$

**Correct option is (d)**

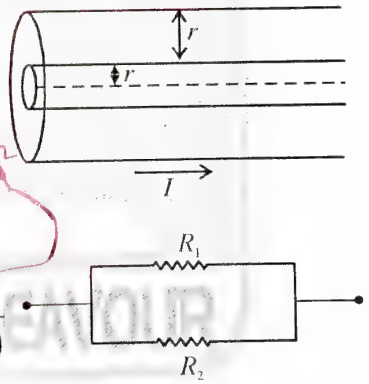
28. We can treat the system as parallel combination of two resistance.

Now, $R_1 = \left(\frac{\rho_1 L}{A_1}\right) = \frac{\rho_1 L}{\pi r^2}$

and $R_2 = \frac{\rho_2 L}{A_2} = \frac{\rho_2 L}{3\pi r^2}$

Therefore, the total resistance

$$= \frac{R_1 R_2}{R_1 + R_2} = \frac{\frac{\rho_1 L}{\pi r^2}\times\frac{\rho_2 L}{3\pi r^2}}{\frac{L}{\pi r^2}\left(\rho_1 + \frac{\rho_2}{3}\right)} = \frac{\rho_1\rho_2 L}{\pi r^2(3\rho_1 + \rho_2)}$$




**Correct option is (d)**

29. Given wave function, $\psi(x) = e^{-x/2}$ $0 < x < \infty$ is normalized. We can check the normalization condition.

Therefore, the probability of finding the particle between $x = 0$ to $x = \ln(2)$ is

$$P = \int_0^{\ln(2)} |\psi(x)|^2\,dx = \int_0^{\ln(2)} e^{-x}\,dx = \left[\frac{e^{-x}}{-1}\right]_0^{\ln(2)} = \left(-\frac{1}{2}+1\right) = \frac{1}{2}$$

**Correct option is (b)**

30. Given : $I = \iiint (x^2+y^2)\,d^3r = \iiint (x^2+y^2)\,dxdydz$

$= \iint (x^2+y^2)[z]_0^{(4-x^2-y^2)}\,dxdy = \iint (x^2+y^2)(4-x^2-y^2)\,dxdy$

[Now, take $x = r\cos\theta$, $y = r\sin\theta$ ]

$$=\int_0^1\int_0^{2\pi} r^2\left(4-r^2\right)rdrd\theta=\left[r^4-\frac{r^6}{6}\right]_0^1\left[\theta\right]_0^{2\pi}=\left(1-\frac{1}{6}\right)2\pi=\frac{5\pi}{3}$$

**Correct option is (a)**

31. Given wave function: $\phi(x)=\alpha\psi_0(x)+\beta\psi_1(x)$

According to normalization condition, $\langle\phi(x)|\phi(x)\rangle=1 \Rightarrow \alpha^2+\beta^2=1$ ... (i)

We know the position operator, $\hat{x}=\sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}+\hat{a}^\dagger\right)$

$$\therefore \quad \langle x\rangle=\langle\psi|\hat{x}|\psi\rangle=\left(\langle\psi_0|\alpha+\langle\psi_1|\beta\right)\sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}+\hat{a}^\dagger\right)\left(\alpha|\psi_0\rangle+\beta|\psi_1\rangle\right)$$

$$=\left(\langle\psi_0|\alpha+\langle\psi_1|\beta\right)\sqrt{\frac{\hbar}{2m\omega}}\left[0+\beta|\psi_0\rangle+\alpha|\psi_1\rangle+\sqrt{2}\beta|\psi_2\rangle\right]$$

$$=\sqrt{\frac{\hbar}{2m\omega}}(2\alpha\beta) \qquad \left[\because\langle\psi_m|\psi_n\rangle=\delta_{mn}\right]$$

The product of $\alpha$ and $\beta$ i.e. $\alpha\beta$ will be minimum if $\alpha$ and $\beta$ have opposite signs. Therefore, $\langle x\rangle$ will be minimum when $\alpha$ and $\beta$ have opposite signs.

From equation (1), $\alpha^2+(-\alpha)^2=1 \Rightarrow 2\alpha^2=1 \Rightarrow \alpha=\pm\frac{1}{\sqrt{2}}$. So, $\beta=-\alpha=\mp\frac{1}{\sqrt{2}}$

**Correct option is (a)**

32. Given wave function: $\psi(x)=\left(\frac{x}{a}\right)^n\exp\left(-\frac{x}{a}\right)$

General form of time-independent schrodinger equation,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi(x)=E\psi(x)$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\left[\frac{d}{dx}\left(\left(n\frac{x^{n-1}}{a^n}\right)\exp\left(-\frac{x}{a}\right)-\frac{1}{a}\left(\frac{x}{a}\right)^n\exp\left(-\frac{x}{a}\right)\right)z\right]+V(x)\psi(x)=E\psi(x)$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\left[n(n-1)\frac{x^{n-2}}{a^n}\exp\left(-\frac{x}{a}\right)-\frac{nx^{n-1}}{a^{n+1}}\exp\left(-\frac{x}{a}\right)-\frac{nx^{n-1}}{a^{n+1}}\exp\left(-\frac{x}{a}\right)+\frac{x^n}{a^{n+2}}\exp\left(-\frac{x}{a}\right)\right]$$
$$+V(x)\psi(x)=E\psi(x)$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\left[\frac{n(n-1)}{x^2}-\frac{2n}{ax}+\frac{1}{a^2}\right]\psi(x)+V(x)\psi(x)=E\psi(x)$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\left[\frac{n(n-1)}{x^2}-\frac{2n}{ax}+\frac{1}{a^2}\right]+V(x)=E$$

$$\Rightarrow \quad V(x)=\frac{\hbar^2}{2m}\left[\frac{n(n-1)}{x^2}-\frac{2n}{xa}\right]+\left(E+\frac{\hbar^2}{2ma^2}\right)$$

Given: $V(x)\to 0,\ x\to\infty$

$$\therefore \quad E+\frac{\hbar^2}{2ma^2}=0 \quad \text{and} \quad V(x)=\frac{\hbar^2}{2m}\left(\frac{n(n-1)}{x^2}-\frac{2n}{xa}\right)$$

**Correct option is (d)**

33. Let $\phi(x,y,z)=0$ represents the equation of the surface.

Given: $x=4\alpha^2+\beta;\ y=\sin\beta;\ z=\cos\alpha$

$$\Rightarrow \quad x=4\left(\cos^{-1}z\right)^2+\sin^{-1}y \Rightarrow \phi(x,y,z)=x-4\left(\cos^{-1}z\right)^2-\sin^{-1}y=0$$

The equation of the tangent to the surface $\phi$ at $\left(\pi^2,0,0\right)$ is

$$\Rightarrow \quad \left.\frac{\partial\phi}{\partial x}\right|_{(x_0,y_0,z_0)}(x-x_0)+\left.\frac{\partial\phi}{\partial y}\right|_{(x_0,y_0,z_0)}(y-y_0)+\left.\frac{\partial\phi}{\partial z}\right|_{(x_0,y_0,z_0)}(z-z_0)=0$$

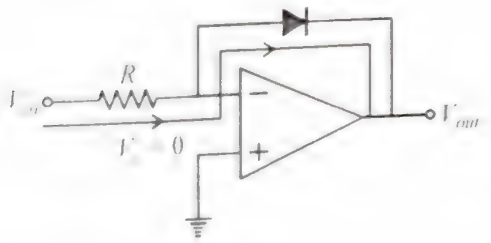
$$\Rightarrow \quad 1\Big|_{(\pi^2,0,0)}\cdot\left(x-\pi^2\right)-\left.\frac{1}{\sqrt{1-y^2}}\right|_{(\pi^2,0,0)}(y-0)-8\cos^{-1}z\left.\frac{(-1)}{\sqrt{1-z^2}}\right|_{(\pi^2,0,0)}(z-0)=0$$

$$\Rightarrow \quad \left(x-\pi^2\right)-\frac{1}{1}\times y-8\times\frac{\pi}{2}(-1)z=0$$

$$\Rightarrow \quad x-y+4\pi z=\pi^2$$

**Correct option is (d)**

34. The group velocity of wave in any medium will always be less than the speed wave light in free space.
But there is no such restriction on phase velocity. So, it can take any value.
**Correct option is (b)**

35. 


Applying KVL at A

$$\frac{V_a-V_{in}}{R}+I_0\left(e^{-\frac{eV_0}{nk_BT}}-1\right)=0$$

$$\Rightarrow \quad 0-\frac{V_{in}}{R}=-I_0\,e^{\frac{eV_0}{nk_BT}}$$

$$\Rightarrow \quad V_0 = \frac{e}{\eta k_B T} \ln\left(\frac{V_{in}}{I_0 R}\right)$$

Since, $V_0 \propto \ln(V_{in})$

The amplifier is a logarithmic amplifier

**Correct option is (c)**

36. We know that,

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{(2n+1)}}{2n+1}$$

**Correct option is (d)**

37. Here, Number of slits are N = 3.

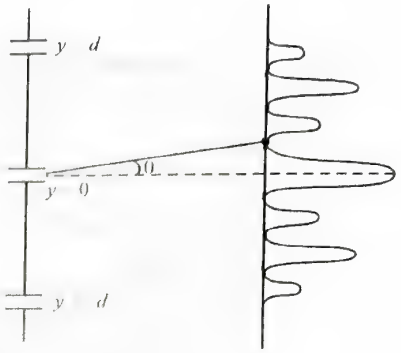
Position of secondary minima of diffraction pattern

$$d \sin\theta = \frac{P\lambda}{N} = \frac{P\lambda}{3}$$

Position of first minima, P = 1

$$\sin\theta = \frac{\lambda}{3d} \Rightarrow \theta = \sin^{-1}\frac{\lambda}{3d}$$

**Correct option is (a)**




38. Let's assume $A$ to be a dimensionless constant. Then, from dimension analysis

$$[M] = [\rho]^{\alpha} [v]^{\beta} [g]^{\gamma}$$

$$\Rightarrow \quad [M] = \left[ML^{-3}\right]^{\alpha} \left[LT^{-1}\right]^{\beta} \left[LT^{-2}\right]^{\gamma}$$

$$[M] = [M]^{\alpha} [L]^{-3\alpha+\beta+\gamma} [T]^{-\beta-2\gamma}$$

$\therefore \quad \alpha = 1, \; -3\alpha + \beta + \gamma = 0$ and $-\beta - 2\gamma = 0$

$\therefore \quad \beta + \gamma = 3\alpha = 3$

Now, $\beta = -2\gamma$

$\therefore \quad -2\gamma + \gamma = 3 \Rightarrow \gamma = -3$

$\therefore \quad \beta = 6$

$\therefore \quad M = \frac{A\rho v^6}{g^3}$

**Correct option is (b)**

39. Given wavefunction: $\psi(x) = A \exp\left(-\frac{x^2}{2\alpha^2} + ikx\right)$

The expectation value of momentum of the particle is given by

$$\langle p\rangle = \frac{\langle\psi|\hat{p}|\psi\rangle}{\langle\psi|\psi\rangle} = -ih\frac{\left\langle\psi\left|\frac{d}{dx}\right|\psi\right\rangle}{\langle\psi|\psi\rangle} = -ih\frac{\int_{-\infty}^{\infty}\psi^*\frac{d\psi}{dx}dx}{\int_{-\infty}^{\infty}\psi^*\psi\,dx}$$

$$= -ih\frac{\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{2\alpha^2}-ikx\right)\left(-\frac{x}{\alpha^2}+ik\right)\exp\left(-\frac{x^2}{2\alpha^2}+ikx\right)dx}{\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{\alpha^2}\right)dx}$$

$$= -ih\frac{\int_{-\infty}^{\infty}\left(-\frac{x}{\alpha^2}+ik\right)\exp\left(-\frac{x^2}{\alpha^2}\right)dx}{\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{\alpha^2}\right)dx} = -ih\frac{-\frac{1}{\alpha^2}\int_{-\infty}^{\infty}x\exp\left(-\frac{x^2}{\alpha^2}\right)dx + ik\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{\alpha^2}\right)dx}{\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{\alpha^2}\right)dx}$$

Integrand in the first part of the numerator, is an odd function. Therefore, the value of the integral will be zero.

$$= -ih\frac{0+ik\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{\alpha^2}\right)dx}{\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{\alpha^2}\right)dx} = \hbar k$$

**Correct optino is (b)**

40. Reduced mass $\mu = \frac{m^2}{2m} = \frac{m}{2}$

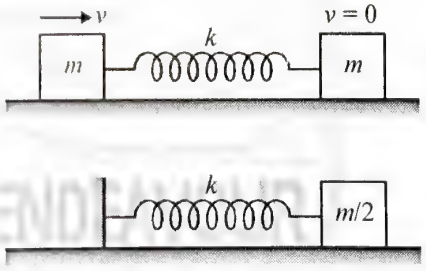


Fig. (1)

$$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k}{m/2}} = \sqrt{\frac{2k}{m}}$$



Equivalent system

Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

The time when the second block will come to rest is when the original configuration is repeated as shown in figure (1),

$$t = T = 2\pi\sqrt{\frac{m}{2k}} = \pi\sqrt{\frac{2m}{k}}$$

**Correct option is (a)**

41. Given: $\psi(x) = \begin{cases} 2\alpha\sqrt{\alpha}\,x\,e^{-\alpha x} & x > 0 \\ 0 & x < 0 \end{cases}$

Therefore, the probability density corresponding to the given wavefuntion:

$$P(x) = |\psi(x)|^2 = \begin{cases} \left(4\alpha^3 x^2 e^{-2\alpha x}\right) & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

For maxima or minima of $P(x)$,

$$\frac{dp}{dx}=0 \Rightarrow \quad 2x\,e^{-2\alpha x}+x^2\left(-2\alpha\right)e^{-2\alpha x}=0 \Rightarrow \quad x=\frac{1}{\alpha}$$

We can check that $\left.\frac{d^2P}{dx^2}\right|_{x=\frac{1}{\alpha}}<0$

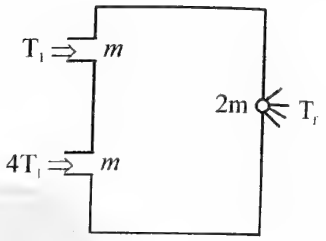
Therefore, it will correspond to a maxima.

**Correct option is (c)**

42. The amount of heat in take of water is

$$\Delta Q=m\left(4T_1-T_f\right)-mC\left(T_f-T_1\right)$$

$$=mC\left(5T_1-2T_f\right)$$




As the machine is in steady state, we have

$$\frac{1}{2}(2m)v^2=\Delta Q=mC\left(5T_1-2T_f\right)$$

$$\Rightarrow \quad v=\sqrt{C\left(5T_1-2T_f\right)}$$

Since, the increase in entropy is always positive, we have,

$$\Delta S=mC\ln\left(\frac{T_f}{T_1}\right)+mC\ln\left(\frac{T_f}{4T_1}\right)\geq 0$$

$$\Rightarrow \quad T_f\geq 2T_1$$

$$\therefore \quad v_{max}=\sqrt{C\left(5T_1-4T_1\right)}=\sqrt{CT_1}$$

**Correct option is (a)**

43. $A_{(m)} \longrightarrow B_m + C_{(massless)}$

$M \longrightarrow m + 0$

$rest \qquad p_B + p_C$

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Now, from momentum conservation $|p_B|=|p_C|$

$$\Rightarrow \quad m_Ac^2=\sqrt{m_B^2c^4+p_B^2c^2}+\sqrt{p_C^2c^2}$$

$$\Rightarrow \quad \left(m_Ac^2-p_Cc\right)=\sqrt{m_B^2c^4+p_B^2c^2} \qquad \left(\because p_B=p_C\right)$$

On squaring, we get

$$m_A^2c^4+p_C^2c^2-2m_Ap_Cc^3=m_B^2c^4+p_B^2c^2$$

$$\Rightarrow \quad \left(m_A^2c^4-m_B^2c^4\right)=2m_Ap_Cc^3$$

$$\Rightarrow \quad \frac{\left(M^2-m^2\right)c}{2M}=p_C \Rightarrow \frac{\left(M^2-m^2\right)c}{2M}=p_C$$

**Correct option is (b)**

44. Given: $\mu = 58\ kg;\ \sigma = 5\ kg$

$a = \mu - \sigma = 53\ kg$

$b = \mu + \sigma = 63\ kg$

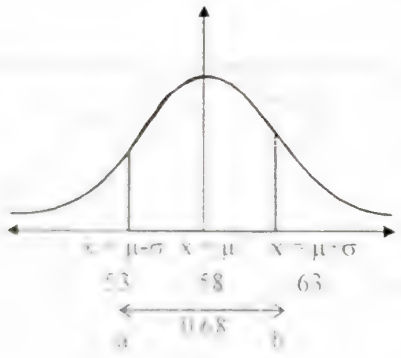
Since, the area under the probability density curve represents the probability of finding x, therefore

$$P(\mu - \sigma \le x \le \mu + \sigma) = 0.68(\text{approx})$$

So, the number of students having weight in the range 53 to 63 kg is

$= 100 \times$ area under the curve from a to b $= 100 \times 0.68 = 68$

(Since, the area within one standard deviation of a normal distribution is 0.68; See the figure below)




**Correct option is (b)**

45. N number of fringes appear on the screen when

$$2L = N\lambda \qquad \ldots (i)$$

Therefore, (N + 100) number of fringes appear in the screen

$$2\mu L = (N + 100)\lambda \qquad \ldots (ii)$$

From equation (i) and (ii), we get

$$(\mu - 1) = \frac{100\lambda}{2L} = \frac{100 \times 632.2 \times 10^{-9}}{2 \times 10 \times 10^{-2}} = 36.1 \times 10^{-5}$$

Therefore, the refractive index of the medium is $\mu = 1.00032$.

**Correct option is (c)**

46. Let $\theta$ be the angle made by the rope w.r.t. vertical.

Torque equation of the child,

$$\tau = -mgl - 4mgl\sin\theta$$

$$\Rightarrow \quad 4ml^2\alpha = -mgl - 4mgl\sin\theta$$

$$\alpha = -\frac{g}{4l} - \frac{g}{l}\sin\theta \quad (\text{for } \theta \approx 0)$$

Using, $\omega^2 = \frac{g}{l}$ and assuming $\sin\theta = \theta$

$$\alpha + \omega^2\theta = -\frac{\omega^2}{4}$$

General solution:

$$\theta(t) = A\cos\omega t + B\sin\omega t - \frac{1}{4}$$

At $t = 0$, $\theta = \frac{1}{2}$ and $\dot{\theta} = 0$

$$\Rightarrow \quad \frac{1}{2} = A - \frac{1}{4} \Rightarrow A = \frac{3}{4}$$

Also, $\dot{\theta} = -A\omega\sin\omega t + B\omega\cos\omega t$

$$\Rightarrow \quad 0 = B\omega \Rightarrow B = 0$$

$$\therefore \quad \theta(t) = \frac{3}{4}\cos\omega t - \frac{1}{4}$$

$$\Rightarrow \quad \frac{3}{4}\cos\omega t_1 = \frac{1}{4} \Rightarrow \cos\omega t_1 = \frac{1}{3}$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{\ell}{g}}\cos^{-1}\left(\frac{1}{3}\right)$$

**Correct option is (*)**

47. The condition of Bragg diffraction $2d\sin\theta = n\lambda$

$$\Rightarrow \quad \lambda = \left(\frac{2d\sin\theta}{n}\right)$$

$$\Rightarrow \quad \ln\lambda = \ln 2 + \ln d + \ln\sin\theta + \ln n$$

$$\Rightarrow \quad \left|\frac{\Delta\lambda}{\lambda}\right| = \left|\frac{\Delta d}{d}\right| + \cot\theta\,|\Delta\theta|$$

Now the volume of a unit cell is $d^3 = \frac{M}{\rho N_0}$

$$\Rightarrow \quad \left|\frac{\Delta d}{d}\right| = \frac{1}{3}\left|\frac{\Delta\rho}{\rho}\right|$$

$$\therefore \quad \left|\frac{\Delta\lambda}{\lambda}\right| = \frac{1}{3}\left|\frac{\Delta\rho}{\rho}\right| + \cot\theta\,|\Delta\theta|$$

$$= \frac{1}{3}\times 10^{-2} + \left|\frac{\Delta\theta}{\theta}\right|$$

$$= 0.33\times 10^{-2} + \left(\frac{3}{6\times 60}\right)$$

$$= 0.33\times 10^{-2} + 0.0083$$

$$= 1.1\times 10^{-2}$$

**Correct option is (b)**

48. The time-independent schrodinger equation of a particle which is confined to move on a circle of radius $a$ is

$$-\frac{h^2}{2m}\frac{1}{a^2}\frac{d^2\psi}{d\theta^2}=E\psi \quad \Rightarrow \quad \frac{d^2\psi}{d\theta^2}=-\frac{2ma^2E}{h^2}\psi(\theta) \quad \Rightarrow \frac{d^2\psi}{d\theta^2}+n^2\psi=0$$

General solution for the wave fuction of the particle, will be

$$\psi = A\,e^{in\theta}+Be^{-in\theta}\left(n=\sqrt{\frac{2mEa^2}{h^2}}\right)$$

Therefore, the normalized wave function of the particle will be

$$\psi=\frac{1}{\sqrt{2\pi}}e^{in\theta}\ (n=\text{integer})$$

The energy eigenvalues of the particle are $=\dfrac{h^2n^2}{2ma^2}$

Now, in presence of magnetic field an additional energy $=\vec{\mu}\cdot\vec{B}=\pm\mu B$ will be added.

Therefore, total energy eigenvalues of the particle will be $E=\dfrac{h^2n^2}{2ma^2}\pm\mu B$

**Correct option is (d)**

49. The wavelength shift (Compton shift) due to scattering by angle $\phi$ is $\Delta\lambda=\dfrac{h}{m_ec}(1-\cos\phi)$

In first collision, angle of scattering is $90^0$. So, after first scattering the wavelength shift will be

$$=\frac{h}{m_ec}(1-\cos 90^\circ)=\frac{h}{m_ec}$$

In second collision, angle of scattering is $60^0$. So, after second scattering the wavelength shift will be

$$=\frac{h}{m_ec}(1-\cos 60^\circ)=\frac{h}{m_ec}\left(1-\frac{1}{2}\right)=\frac{h}{2m_ec}$$

Therefore, the wavelength of the final photon is

$$\lambda=\lambda_0+\frac{h}{m_ec}+\frac{h}{2m_ec}=\lambda_0+\frac{3}{2}\left(\frac{h}{m_ec}\right)$$

$$=\left(0.4+\frac{3}{2}\times 0.0243\right)\text{Å}=(0.4+0.0363)\text{Å}=(0.4363)\text{Å}$$

**Correct option is (b)**

50. B = 0.3 T

$\lambda = 450\ nm = 450\times10^{-9}m$

$$\therefore \quad \Delta\lambda=\frac{eB\,\lambda^2}{4\pi mc}=\frac{1.6\times10^{-19}\times0.3T\times\left(450\times10^{-9}\right)^2}{4\times3.14\times9.11\times10^{-31}\times3\times10^8}$$

$$=283.16\times10^{-14}=0.00283\times10^{-9}m\cong0.00283\ nm$$

**Correct option is (c)**